Study Guide for Exam 1

Overall, make sure you understand the topics and problems below and all homework problems.

Algebraic Reasoning and Representation

• Know the four uses for a variable below and be able to identify them or illustrate them with an example.
  1. Describe generalized properties
  2. Express relationships
  3. Serve as unknowns
  4. Express formulas

• Understand the distinction between variables, constants, algebraic expressions, and equations.

• For an equation, be able to identify and give the definition of the domain, solution set, and equivalent equations.

• Be able to give the definition of a function and find the domain and range for a given function.

• Know the five descriptions of functions below and be able to identify them or illustrate them with an example. Be able to discuss how they would be helpful or how you would use them in your classroom.
  1. as formulas
  2. as tables
  3. as arrow diagrams
  4. as function machines
  5. as graphs

• Understand a Cartesian coordinate system and it’s quadrants.

• Understand slope and what it means for lines to be parallel.

• Be able to state and apply the distance formula.

• For the three equations of a line below, be able identify each and give examples. Be able to convert from one to another and be able to write problems using a given form.
  1. point-slope form
2. slope-intercept form
3. two-point form

Statistics

- Know the seven graphical representations of data below and be able to identify them or illustrate with an example. Be able to discuss what type of data is appropriate for each representation.
  1. Dot Plots
  2. Stem and Leaf Plots
  3. Histograms
  4. Line Graphs
  5. Bar Graphs
  6. Pie Charts
  7. Pictographs

- Be able to compute each of the three measures of central tendency and explain for what sort of data it would or would not be useful.
  1. mean
  2. median
  3. mode

- Be able to compute each of the measures of variability and interpret the results.
  1. range
  2. upper quartile, lower quartile, and IQR
  3. standard deviation

- Be able to find and define outliers.

- Understand the 5-number summary and be able to draw and interpret a box plot.

- Be able to discuss the difference between a population and a sample. Be able to generate a random sample and explain why it is random.

- Understand a normal distribution, relative frequency, and the 68-95-99.7 Theorem. Be able to give examples.

- Understand a standardized distribution or z-curve. Be able to compute the z-score.

- Be able to compute percentile.
Exercises

1. Find the equation of each of the lines below. State the form you use.
   
   (a) The line through (3, 4) with slope 2.
   (b) The line through (6, −1) and (−2, 5).
   (c) The line of slope 3 that intersects the y-axis at y = −4.

2. Prove that (1, 2), (7, 10), and (5, −1) are the vertices of a right triangle.

3. For \( x = ky + 5 \), determine \( k \) so the line is perpendicular to \( 2y = −6x + 7 \).

4. Draw approximate sketches of graphs that correspond to the following functions of time.
   
   (a) Temperature of a forgotten cup of coffee
   (b) Perceived pitch of a train whistle as the train passes
   (c) Height of the water in a bathtub, during the time someone takes a bath
   (d) Hours of daylight in Chicago during a calendar year (domain= \{1, 2, \ldots, 365\})

5. Some children are in line and each is assigned one operation to perform. If the first child writes \( x \) on a sheet of paper and the last child is handed a sheet with the expression \((3−5x)^3+4\), describe how many children are in the line and what order each operation is performed.

6. Little Red Riding Hood rode her bike 15 miles to Grandmother’s house, arriving in 3 hours. When she discovers the Big Bad Wolf, she rides home at a rate of 15 miles per hour.
   
   (a) What was Little Red Riding Hood’s average speed to Grandmother’s house?
   (b) How long did it take her to get home?
   (c) How many hours did the round trip take?
   (d) What is the average speed for the round trip?
   (e) Is this answer in part (c) equal to the average of the speed to and from Grandmother’s house? Why or why not?
   (f) Draw a graph representing Little Red Riding Hood’s trip, with distance on the \( y \)-axis and time on the \( x \)-axis.

7. Toss seven pennies 20 times and record the number of heads each time.
   
   (a) Draw a relative-frequency polygon (i.e. a line graph with relative-frequency on the \( y \)-axis).
   (b) Calculate the mean and the standard deviation.
(c) Determine what percent of the data lies within one standard deviation of the mean.

(d) Determine what percent of the data lies within two standard deviations of the mean.

(e) Does the $68 - 95 - 99.7$ Rule hold for this population?

8. Given the data, \{18, 27, 17, 19, 21, 24, 18, 15\},

(a) Compute the mean, median, and mode.

(b) Find $Q_L$ and $Q_U$. Determine the IQR.

(c) Are there any outliers?

(d) Give the 5-number summary for the data and draw a box plot.

9. Suppose you generate a random sequence of 0s and 1s by repeatedly rolling a die and recording a 0 if an even number come up and a 1 if an odd number comes up. Is this a random sequence of 0s and 1s? Explain.

10. For a population with a normal distribution with mean 45 and with standard deviation 5,

(a) about 68% of the population lies within what limits?

(b) about 95% of the population lies within what limits?

(c) about 99.7% of the population lies within what limits?

(d) Compute the $z$-scores for sample points 33 and 46.

(e) Compute the percentile for each sample point in part (d).

11. Suppose you have a set of data representing yearly income for people in Kentucky. Assume it is normally distributed. Would you rather the $z$-score for your data point be -1 or 0.5? Why?