1. (9 points) Suppose 72 runners finish a marathon.
   
   (a) How many ways could the 72 runners finish the race?

   (b) If gold, silver, and bronze medals are awarded for 1st, 2nd, and 3rd place respectively, how many ways could the medals be distributed?

   (c) If the top 10 finishers receive a free shirt, how many ways could the shirts be distributed without regard to order?
2. (a) (16 points) Give the definition of a convex polygon.

(b) Draw a convex heptagon.

(c) Can you draw a concave quadrilateral? Why or why not?

(d) Can you draw a concave $n$-gon for any $n \geq 3$? Why or why not?
3. (12 points) Assume the normal 26 letter alphabet.

(a) How many code words of length 4 can be formed \textbf{without} repetition?

(b) How many code words of length 4 can be formed \textbf{without} repetition and including one “a”?

(c) How many code words of length 4 can be formed \textbf{allowing} repetition and including at least one “a”?
4. (a) (8 points) What is the sum of measures of all interior angles of an $n$-gon?

(b) What is the measure of one interior angle of a regular $n$-gon?

(c) Explain how you would prove to your students that this formula is true.
5. (6 points) Your clothes have just come out of the washing machine and you are hanging your socks on the clothes line. Suppose you have 4 blue socks, 6 yellow socks, and 3 red socks. Assuming the socks are indistinguishable except for color, in how many ways can you hang the socks on the line? Explain how you got your answer.

6. (5 points) Suppose the empirical probability of rolling an even number on one die is 0.4 but the theoretical probability of rolling an even number is 0.5. How can it be that these numbers are not the same? What is the difference between empirical probability and theoretical probability?
7. (12 points) Your student Amanda is shown the following picture of a box.

You ask Amanda to tell you how many faces, edges, and vertices the box has. She says, “The box has 3 faces, 9 edges, and 7 vertices”.

(a) What does Amanda understand correctly?

(b) What is Amanda doing incorrectly?

(c) How would you help her understand her mistake?
8. (12 points) Suppose there exist a polyhedron with 4 vertices and 6 edges.

(a) How many faces does this polyhedron have? How do you know?

(b) Draw a net diagram of this polyhedron, that is a diagram of the polyhedron unfolded. What is the name of this polyhedron?

(c) How does a net diagram of a polyhedron help your students understand the definition of polyhedron?
9. (20 points) On your way home, an evil and not so smart troll captures you and forces you to play the following game. He is thinking of two different numbers, without order, between 1 and 5. He says if you guess both the numbers correctly, he will give you $30. If you guess exactly one of the numbers correctly, he will give you $10. If you guess both incorrectly, he will take $20.

(a) What is the probability of guessing both numbers correctly?

(b) What is the probability of guessing exactly one number correctly?

(c) What is the probability of guessing both numbers incorrectly?

(d) What is the expected value of this game? Find an exact answer.

(e) If you had a choice would you play this game? Why?