Study Guide for Exam 1

To study for the test, I recommend reading the chapter reviews in chapters 1 and 2 and reviewing your notes and homework problems. I’ve listed problems you should be able to solve, and more good problems to study can be found after the chapter reviews in the textbook.

1. Consider the following sentence which is true in Euclidean geometry: “If a polygon has 5 sides, then its interior angles measure $540^\circ$.”
   (a) State the converse. Is the converse true?
   (b) State the inverse. Is the inverse true?
   (c) State the contrapositive. Is the contrapositive true?

2. Give an example of an “If..., then ...” sentence about quadrilaterals that is true such that its converse is also true.

3. Find the perimeter of a triangle lying in $xyz$-space with coordinates $(2, 5, 0), (-1, 1, 1),$ and $(4, 0, 0)$.

4. In the $xy$-plane, let $P$ be the point with coordinates $(1, 3)$ and let $Q$ be the point with coordinates $(4, -2)$.
   (a) Find the equation of the line through $P$ and $Q$ in the form $Ax + By + C = 0$.
   (b) Find the equation of the line through $P$ and $Q$ in the form $y = mx + b$.
   (c) Find the intersection of this line and the $x$-axis.

5. Let $S = \{-2, -1, 0, 1, 2\}$. Define a relation on $S$ by $a \sim b$ if $a + b > 0$. Check if this relation is reflexive, symmetric, and transitive. Is it an equivalence relation?

6. State Euler’s formula for polyhedra. Give an example of a three dimensional object for which this formula does NOT hold.

7. Find the number of edges in a polygon 38 faces and 24 vertices.

8. Name the regular polyhedra. Which are dual to each other?

9. Two people are standing on the Earth with the angle between them from the center of the Earth measuring $25^\circ$. Approximate the radius of the Earth with 4,000 miles. How far apart are the people standing?

10. The triangle $\triangle ABC$ has $AB = BC = 5$ and $AC = 8$.
    (a) Which two angles have the same measure? Explain your reasoning.
    (b) Is the third angle larger or smaller that the two angles having the same measure? Explain.
11. Explain how you would derive the area formula for
   (a) a rectangle
   (b) a parallelogram
   (c) a triangle
   (d) a trapezoid

12. List 3 different criteria that can be used to determine if a figure is a parallelogram. (Hint: Look in Section 2.3)

13. Prove each of the following statements.
   (a) The perpendicular bisector of a chord passes through the center of a circle.
   (b) A quadrilateral is a parallelogram iff each pair of consecutive angles are supplementary.
   (c) Let two lines $l_1$ and $l_2$ be parallel and let $l_3$ be a transversal that is perpendicular to the first line $l_1$, then $l_3$ is also perpendicular to $l_2$.

14. Do not approximate $\pi$. Find the volume of
   (a) a right circular cone with a base with a radius of 3 cm and a height of 7 cm
   (b) a hemisphere with radius 4 ft
   (c) a right rectangular prism with dimensions 6 ft $\times$ 2 ft $\times$ 5 ft

15. Do not approximate $\pi$. Find the surface areas for each object in the previous question.

   The following essay question will be on the test. I expect a well written answer longer than just a few sentences.

   1. Your student says, “I don’t understand how two rectangles with exactly the same perimeter can enclose different areas. Can you explain this to me?” (page 61 in the text). Write your explanation in complete sentences.