

## Study Guide for Exam 2

To study for the test, I recommend reading the chapter reviews in chapters 3 and 4 and reviewing your notes and homework problems. I've listed problems you should be able to solve, and more good problems to study can be found after the chapter reviews in the textbook.

**Note: There will be NO calculators allowed on the exam.**

1. Is the set of odd numbers a group with the operation of addition? Why or why not?
2. Is the set of even numbers a group with the operation of addition? Why or why not?
3. Let  $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the transformation given by  $F(x, y, z) = (x, y - 2, z + 3)$ .
  - (a) Find  $F(1, -1, 0)$ ,
  - (b) Is  $F$  an isometry? Prove or disprove.
4. Let  $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the transformation given by  $F(x, y) = (-3y, x)$ .
  - (a) Find  $F(-5, -6)$ .
  - (b) Is  $F$  an isometry? Prove or disprove.
5. Let a triangle  $XYZ$  satisfy  $XY = 4\text{in}$ ,  $XZ = 6\text{in}$ , and  $m(\angle X) = 35^\circ$ . Find  $YZ$  up to one decimal place.
6. Let a triangle  $EFG$  satisfy  $EF = 3\text{cm}$ ,  $FG = 7\text{cm}$ , and  $EG = 6\text{cm}$ . Find the measure of angles  $E$ ,  $F$ , and  $G$ .
7. Find the group of symmetries of a square.
8. Find the group of symmetries for a capital I. Write the multiplication table for this group.
9. Let  $QRS$  be a triangle with vertices  $Q = (1, 1)$ ,  $R = (-1, 1)$ , and  $S = (0, 0)$ . Is  $QRS$  a right triangle? Why or why not?
10. Let  $A = (1, 1)$ ,  $B = (3, 3)$  and  $C = (1, 3)$ .
  - (a) Find the image of the triangle  $ABC$  under the reflection  $r_l$  where  $l$  is the line  $y = x$ .
  - (b) Find the image of the triangle  $ABC$  under the reflection  $r_h$  where  $h$  is the  $y$ -axis.
  - (c) Find the image of the triangle  $ABC$  under the composition  $r_h \circ r_l$ .
11. Let  $Q = (0, 1)$ ,  $R = (-1, 4)$  and  $C = (-2, 0)$ .
  - (a) Find the image of the triangle  $QRS$  under the rotation  $R_{C, \alpha}$  where  $C$  is the origin and  $\alpha = -90^\circ$ .

- (b) Find the image of the triangle  $QRS$  under the rotation  $R_{C,\beta}$  where  $C$  is the origin and  $\beta = 180^\circ$ .
12. Let  $G_1$  and  $G_2$  be two similar polygons. Suppose the area of  $G_1$  is twice the area of  $G_2$ .
- (a) Find the scale factor  $S$  for  $G_1 \sim G_2$ .
- (b) If a side in  $G_2$  has length 3in, find the length of the corresponding side in  $G_1$
13. You have two similar right circular cylinders  $C_1$  and  $C_2$ . The lateral surface area of  $C_2$  is 144 times the lateral surface area of  $C_1$ . Find the height of  $C_1$  if the height of  $C_2$  is 24in.
14. Give proofs for the following:
- (a) Law of Cosines
- (b) Pythagorean Theorem
- (c) The composition of any two reflections across intersecting lines is a rotation.
- (d) The composition of any two reflections across parallel lines is a translation.
15. Let  $ABCD$  be the rectangle with vertices  $(-1, 0)$ ,  $(-1, 5)$ ,  $(2, 0)$ , and  $(2, 5)$  respectively. Graph the image of  $ABCD$  under a dilation with center  $(0, 0)$  and dilation factor 5.
16. This question will be on the test.  
 We defined two sets  $T_1$  and  $T_2$  to be **similar** with scale factor  $S$  if there is a 1-1 correspondence between them such that whenever  $A$  and  $B$  in  $T_1$  correspond to  $A'$  and  $B'$  in  $T_2$  the ratio  $A'B'/AB$  is equal to the fixed number  $S$ . Discuss how this definition can be used to define **congruence** for sets.