

Homework - June 12

Section 1.4 and 1.5

Section 1.4

$$16. \begin{bmatrix} 1 & -3 & -4 & b_1 \\ -3 & 2 & 6 & b_2 \\ 5 & -1 & -8 & b_3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & 3b_1 + b_2 \\ 0 & 14 & 12 & -5b_1 + b_3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & 3b_1 + b_2 \\ 0 & 0 & 0 & 6b_1 - 3b_2 + b_3 \end{bmatrix}.$$

For this matrix to be consistent, we have $6b_1 - 3b_2 + b_3 = 0$. The set of all b_1, b_2 , and b_3 which satisfy this equation are the solutions to the matrix equation $A\mathbf{x} = \mathbf{b}$.

Section 1.5

26. If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then the augmented matrix $[A\mathbf{0}]$ has no free variables. This implies there is a pivot position in every column of A , so the augmented matrix $[A, \mathbf{b}]$ has no free variable. Hence, $A\mathbf{x} = \mathbf{b}$ has a unique solution.