Homework - June 13
Section 1.7

2. $\begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & 8 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 8 & 1 & 0 \\ 0 & 5 & 4 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix}$.
We have a pivot position in every row and no free variable. The only solution is the trivial solution, so the the first three columns are linearly independent.

12. $\begin{bmatrix} 2 & -6 & 8 & 0 \\ -4 & 7 & h & 0 \\ 1 & -3 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 4 & 0 \\ 0 & 5 & 16+h & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
This matrix has a free variable. Hence, there is more than just the trivial solution. Any value of h will make this matrix linearly dependent.

28. Note, a 5×7 matrix may have at most 5 pivots, and a set that spans $\mathbb{R}^5$ must have at least 5 vectors. Further, if a matrix has a free variable, the corresponding free column can be written as a linear combination of the pivot columns. This means the dimension of the span of the set of columns of a matrix is determined only by the number of pivot columns. (Including a free column which is a linear combination, does not increase the dimension of the spanning set.) So, we must have five pivot positions.

40. Suppose a $m \times n$ matrix has n pivots. This means there is a pivot in every column, so there are no free variables. If there are no free variables, $A\curvearrowleft =$ cannot have infinitely many solutions. That means it has at most 1 solution.