Homework - June 29
Section 4.1

6. This set is not a subspace of $\mathbb{P}_n$ because it is not closed under addition. $\mathbf{p_1} = a_1 + t$ and $\mathbf{p_2} = a_2 + t$ are in the set, but $\mathbf{p_1} + \mathbf{p_2} = (a_1 + a_2) + 2t$ is not in the set.

16. W is not a vector space. $W = \{a \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ -6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} : a, b \in \mathbb{R}\}$. The zero vector, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, cannot be written in this form.

30. Suppose $c\mathbf{u} = \mathbf{0}$ and $c \neq 0$. To see that $\mathbf{u} = \mathbf{0}$, multiply both sides of the equation by $\frac{1}{c}$. The number $\frac{1}{c}$ exist because $c \neq 0$ and the associative property of scalars allows you to write

$$\frac{1}{c}(c\mathbf{u}) = (\frac{1}{c}c)\mathbf{u} = 1\mathbf{u} = \mathbf{u}.$$

Because $\frac{1}{c}\mathbf{0} = \mathbf{0}$, we have proven the claim.