Homework - July 3
Section 4.3

10. Augment and reduce to get the matrix $\begin{bmatrix} 1 & 0 & -5 & 0 & 7 & 0 \\ 0 & 1 & -4 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 & -3 & 0 \end{bmatrix}$. The solution set has the form $\mathbf{x} = x_3 \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix}$. Thus, the basis is $\{ \begin{bmatrix} 5 \\ 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -6 \\ 0 \\ 3 \\ 1 \end{bmatrix} \}$.

12. The line contains all points of the form $(x, 5x) = \begin{bmatrix} x \\ 5x \end{bmatrix} = x \begin{bmatrix} 1 \\ 5 \end{bmatrix}$. We have basis $\{ \begin{bmatrix} 1 \\ 5 \end{bmatrix} \}$.

24. Write the vectors of the set in an $n \times n$ matrix A. If the columns are linearly independent, then by the Invertible Matrix Theorem, they also span $\mathbb{R}^n$. Thus, this set is a basis.