Homework - July 10
Section 4.6

2. The rank of A is equal to the number of pivot columns in B which is 3. The dimension of Nul A is $5 - 3 = 2$. A basis for Col A is $\{\begin{bmatrix}1\\-2\\-3\\3\end{bmatrix}, \begin{bmatrix}4\\-6\\-6\\4\end{bmatrix}, \begin{bmatrix}9\\-10\\-3\\0\end{bmatrix}\}$ corresponding to the pivot columns in A. A basis for Row A is $\{\begin{bmatrix}1\\-3\\0\\5\\-7\end{bmatrix}, \begin{bmatrix}0\\0\\2\\-3\\8\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\5\end{bmatrix}\}$ corresponding to the nonzero rows in B. Put B in reduced echelon form $B \sim \begin{bmatrix}1 & -3 & 0 & 5 & 0\\0 & 0 & 1 & -3/2 & 0\\0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 0\end{bmatrix}$ to get the solution set $\mathbf{x} = x_2\begin{bmatrix}3\\1\\0\\0\\0\end{bmatrix} + x_4\begin{bmatrix}-5\\0\\3/2\\1\\0\end{bmatrix}$. A basis for Nul A is $\{\begin{bmatrix}3\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}-5\\0\\3/2\\1\\0\end{bmatrix}\}$.

24. Write such a system in a matrix with 7 rows and 6 columns. Row reductions show that there must be at least one zero row. If we have pivots in every column, it is possible to have a unique solution for any set of right hand coefficients which reduce to a zero in the zero row. It is not possible for all coefficients, because some may leave a row $\begin{bmatrix}0 & 0 & \cdots & 0 & b\end{bmatrix}$.