

Homework - July 16

Section 5.2

2. The characteristic polynomial is $\det \begin{bmatrix} 5-\lambda & 3 \\ 3 & 5-\lambda \end{bmatrix} = (5-\lambda)^2 - 9 = \lambda^2 - 10\lambda + 16 = (\lambda - 8)(\lambda - 2)$. The eigenvalues are 8 and 2.
12. The characteristic polynomial is $\begin{vmatrix} -1-\lambda & 0 & 1 \\ -3 & 4-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda) \begin{vmatrix} -1-\lambda & 0 \\ -3 & 4-\lambda \end{vmatrix} = (2-\lambda)[(-1-\lambda)(4-\lambda) - 0] = (2-\lambda)(-1-\lambda)(4-\lambda)$.
20. Note, $A^T - \lambda I = (A - \lambda I)^T$ and that the characteristic polynomial of A is $\det(A - \lambda I)$. The characteristic polynomial of A^T is $\det(A^T - \lambda I) = \det(A - \lambda I)^T = \det(A - \lambda I)$ because $\det B = \det B^T$ for any matrix B .
24. If A and B are similar, then $A = PBP^{-1}$. $\det(A) = \det(PBP^{-1}) = \det(P)\det(B)\det(P^{-1}) = \det(B)\det(P)\det(P^{-1}) = \det(B)$.