

Homework - July 20

Section 5.3

2. $A^4 = PD^4P^{-1} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1/16 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 151/16 & 90/16 \\ -225/16 & 134/16 \end{bmatrix}.$
8. The eigenvalue of A is $\lambda = 5$ because A is upper triangular. Row reduce $[(A - 5I) \quad \mathbf{0}]$ to $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. A basis for the eigenspace is $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$. This basis is one dimensional, but the multiplicity of 5 is 2. Therefore, A is not diagonalizable.
24. If A is diagonalizable, then for each eigenvalue the dimension of the eigenspace corresponding to that eigenvalue must equal its multiplicity. Because A is 3×3 and has only two distinct eigenvalues, one of these must have multiplicity 2. However, the basis for the eigenspace corresponding to that eigenvalue is one dimensional, so A is not diagonalizable.