Homework - July 24
Section 6.2

8. We check $\mathbf{u_1} \cdot \mathbf{u_2} = -6 + 6 = 0$, so $\mathbf{u_1}$ and $\mathbf{u_2}$ are orthogonal. By theorem, they are also linearly independent. A linearly independent set of size 2 is a basis for $\mathbb{R}^2$. $\mathbf{x} = \frac{\mathbf{x} \cdot \mathbf{u_1}}{\mathbf{u_1} \cdot \mathbf{u_1}} \mathbf{u_1} + \frac{\mathbf{x} \cdot \mathbf{u_2}}{\mathbf{u_2} \cdot \mathbf{u_2}} \mathbf{u_2} = \frac{-18+3}{9+1} \mathbf{u_1} + \frac{12+18}{4+36} \mathbf{u_2} = \frac{-3}{2} \mathbf{u_1} + \frac{3}{4} \mathbf{u_2}$.

16. The distance from $\mathbf{y}$ to the line through $\mathbf{u}$ and $\mathbf{0}$ is given by $||\mathbf{y} - \hat{\mathbf{y}}||$. $\hat{\mathbf{y}} = \frac{-3+18}{1+4} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$. $||\mathbf{y} - \hat{\mathbf{y}}|| = || \begin{bmatrix} -6 \\ 3 \end{bmatrix} || = \sqrt{36 + 9} = 3\sqrt{5}$.

26. A set of nonzero orthogonal vectors is linearly independent. Further, a set of n linearly independent vectors in $\mathbb{R}^n$ forms a basis. The basis for W equals the basis for $\mathbb{R}^n$, therefore $W = \mathbb{R}^n$.