

Homework - July 25
Section 6.3

$$2. \hat{\mathbf{v}} = \frac{\mathbf{v} \cdot \mathbf{u}_1}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1 = \frac{4+10-3+3}{1+4+1+1} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2 \end{bmatrix}. \quad \mathbf{z} = \mathbf{v} - \hat{\mathbf{v}} = \begin{bmatrix} 2 \\ 1 \\ -5 \\ 1 \end{bmatrix}. \quad \text{We have}$$

$$\mathbf{v} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -5 \\ 1 \end{bmatrix}.$$

4. Check $\mathbf{u}_1 \cdot \mathbf{u}_2 = -1 + 1 + 0 = 0$. The orthogonal projection of \mathbf{y} onto

$$\text{Span}\{\mathbf{u}_1, \mathbf{u}_2\} \text{ is } \hat{\mathbf{y}} = \frac{-1+4+0}{1+1+0} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{1+4+0}{1+1+0} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 3/2 \\ 0 \end{bmatrix} + \begin{bmatrix} -5/2 \\ 5/2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}.$$

$$8. \hat{\mathbf{y}} = \frac{-1+4+3}{1+1+1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1+12-6}{1+9+4} \begin{bmatrix} -1 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 3/2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix} \text{ is a vector}$$

in W and $\mathbf{y} - \hat{\mathbf{y}} = \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$ is a vector orthogonal to W . We have $\mathbf{y} =$

$$\begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix} + \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}.$$