Homework - July 27
Section 6.5

2. a) The normal equations are $A^T A\mathbf{x} = A^T\mathbf{b}$. We have
$A^TA = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 8 \\ 8 & 10 \end{bmatrix}$ and $A^T\mathbf{b} = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix} =$
$\begin{bmatrix} -24 \\ -2 \end{bmatrix}$. So, $\begin{bmatrix} 12 & 8 \\ 8 & 10 \end{bmatrix} \mathbf{x} = \begin{bmatrix} -25 \\ -2 \end{bmatrix}$.

   b) To find $\hat{\mathbf{x}}$, we solve $\hat{\mathbf{x}} = (A^TA)^{-1}A^T\mathbf{b} = \begin{bmatrix} 5/28 & -4/28 \\ -4/28 & 6/28 \end{bmatrix} \begin{bmatrix} -24 \\ -2 \end{bmatrix} =$
$\begin{bmatrix} -4 \\ 3 \end{bmatrix}$.

24. When the columns of A are orthogonal, $\hat{\mathbf{x}}$ is given by the weights on the columns of A when we determine the projection of $\mathbf{b}$ onto the column space of A. That is $\hat{\mathbf{x}} = (x_1, x_2, \ldots, x_p) = (\frac{\mathbf{b}\cdot\mathbf{a_1}}{\mathbf{a_1}\cdot\mathbf{a_1}}, \frac{\mathbf{b}\cdot\mathbf{a_2}}{\mathbf{a_2}\cdot\mathbf{a_2}}, \ldots, \frac{\mathbf{b}\cdot\mathbf{a_p}}{\mathbf{a_p}\cdot\mathbf{a_p}})$ where $\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_p}$ are the columns of A. Since the columns are orthonormal, we can simplify this formula to $\hat{\mathbf{x}} = (\mathbf{b}\cdot\mathbf{a_1}, \mathbf{b}\cdot\mathbf{a_2}, \ldots, \mathbf{b}\cdot\mathbf{a_p})$.