Exam 2

MA 322

July 19, 2007

1. Given $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -4 & -6 \\ -3 & 0 & 4 \end{bmatrix}$.

   (a) Find the characteristic polynomial of A.

   (b) Find all eigenvalues of A.

   (c) Choose one eigenvalue λ and find a basis for the eigenspace corresponding to λ.

2. Consider the set $H = \{a_0 + a_1t^2 + a_2t^3 | a_0, a_1, a_2 \in \mathbb{R}\}$. Is H a subspace of $\mathbb{P}_3$? Justify your answer.

3. If λ is an eigenvalue of an invertible matrix A, show λ^{-1} is an eigenvalue of A^{-1}.

4. Given $A = \begin{bmatrix} 1 & 0 & 6 & 2 & -5 \\ 0 & 2 & -1 & 0 & 4 \\ 0 & -6 & 3 & -1 & -5 \end{bmatrix}$

(a) Find a basis for Nul A.

(b) Find a basis for Row A.

(c) Find a basis for Col A.

5. Given $\mathbf{b_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\mathbf{b_2} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$, $\mathbf{d_1} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, and $\mathbf{d_2} = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ and $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}\}$ be bases of a vector space V.

(a) If $[\mathbf{x}]_{\mathcal{D}} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, find the $\mathcal{B}$-coordinate vector of $\mathbf{x}$.

(b) If $[\mathbf{y}]_{\mathcal{B}} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$, find the $\mathcal{D}$-coordinate vector of $\mathbf{y}$.

6. For lunch at a school cafeteria on any given day, students can either choose the salad bar or a hot lunch. Of the students who choose the salad bar today, 30% of them will choose the salad bar tomorrow. Of the students who choose the hot lunch today 80% of them will choose the hot lunch tomorrow.

   (a) Find the stochastic matrix which describes this situation.

   (b) If 20% of the students ate the salad bar yesterday, what percentage of the students are eating the salad bar today?

   (c) What fraction of the students will be eating the hot lunch many days from now?

7. For the following questions, be sure to justify your answer.

(a) Let A be a 3×5 matrix. Suppose all solutions to the homogeneous equation $A\mathbf{x} = \mathbf{0}$ have the form $\mathbf{x} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. What is the rank of A?

(b) Suppose A is an invertible 4×4 matrix. Give an explicit description of Col A.

(c) If A is an $n \times n$ matrix and the kernel of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\{\mathbf{0}\}$, then what is the rank of A?

(d) If A is a 5×7 matrix, can the dimension of the Null space be 1?