

Final Exam

MA 322

August 1, 2007

1. (6 points) Find $(AB)^{-1}$ if $A^{-1} = \begin{bmatrix} 2 & 0 \\ 1 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -1 \\ 6 & -2 \end{bmatrix}$.

2. (10 points) Given $\mathbf{y} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$. Find the distance between \mathbf{y} and $\text{Span}\{\mathbf{u}\}$.

3. (10 points) Determine all values of d so $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ d \end{bmatrix}, \begin{bmatrix} d \\ d+2 \end{bmatrix} \right\}$ is a basis for \mathbb{R}^2 . Explain why these values create a basis.

4. (8 points) Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \\ -5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -6 \\ 9 \\ -8 \\ 3 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 6 \\ -3 \\ 4 \\ -9 \end{bmatrix}$. Find a basis for $H = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Be sure to explain why your answer is a basis.

5. (a) (18 points) Let $A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 3 \\ 0 & 0 & 5 \end{bmatrix}$. Find the eigenvalues for A .

(b) Find a basis for the eigenspace corresponding to each eigenvalue.

(c) Determine if the matrix A is diagonalizable. If it is diagonalizable, find P and D . If it is not diagonalizable explain why.

6. (a) (10 points) Determine the solution set of the homogeneous system

$$\begin{aligned}x_1 - 2x_2 - 9x_3 + 5x_4 &= 0 \\x_2 + 2x_3 - 6x_4 &= 0\end{aligned}$$

Write your answer in parametric vector form.

- (b) If $\mathbf{v} = \begin{bmatrix} -6 \\ 4 \\ 0 \\ -1 \end{bmatrix}$ is a solution to the nonhomogeneous system, find the solution set of the nonhomogeneous system. Write your answer in parametric vector form.

7. (6 points) Show that if \mathbf{x} is in both W and W^\perp , then $\mathbf{x} = \mathbf{0}$.

8. (a) (12 points) Find an orthogonal basis for the subspace spanned by the vectors $\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix}$, and $\mathbf{x}_3 = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$.

(b) Is this basis an orthonormal basis? Why or why not?