

Review Questions for Exam 1

1. Let T be a linear transformation defined by $T(x_1, x_2, x_3) = (x_2 + 3x_3, -x_1)$.

- (a) What is the domain of T ?
- (b) What is the codomain of T ?
- (c) Is T onto? Why?
- (d) Is T one-to-one? Why?
- (e) Is T invertible? Why?

2. Find an LU factorization for $\begin{bmatrix} -5 & 3 & 4 \\ 10 & -8 & -9 \\ 15 & 1 & 2 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 & 0 & -1 \\ 3 & -4 & 3 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 3 \\ 0 & -2 \\ -1 & -1 \\ 5 & 2 \end{bmatrix}$. Find

the following products or explain why they don't exist.

- (a) AB
- (b) BC
- (c) CB
- (d) AC

4. Find the solution set for the homogeneous system $A\mathbf{x} = \mathbf{0}$ where $A =$

$$\begin{bmatrix} 2 & -2 & 1 & 4 \\ 0 & 3 & 2 & -1 \\ 6 & -9 & 0 & 5 \end{bmatrix}.$$

- (a) Write your answer in general form.
- (b) Write your answer in parametric form.

(c) If $\begin{bmatrix} 4 \\ -5 \\ 8 \\ 1 \end{bmatrix}$ is a solution to $A\mathbf{x} = \mathbf{b}$ for some $\mathbf{b} \in \mathbb{R}^3$, what is the solution set for $A\mathbf{x} = \mathbf{b}$?

5. Let A and B be 4×4 matrices such that $\det(A)=2$ and $\det(B) = 3$. Find the following determinants or explain why it is undetermined.

- (a) $\det(AB)$
- (b) $\det(BA^T)$
- (c) $\det(A^4BA^T)$
- (d) $\det(A + B)$

6. Determine if the set of vectors is linearly independent. Justify your answer.

(a) $\begin{bmatrix} 2 \\ 4 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -1 \end{bmatrix}$

7. Determine if the following matrices are invertible. Justify your answers.

(a) $\begin{bmatrix} 2 & 0 & -1 & 5 \\ 3 & 0 & 0 & 2 \\ 5 & 0 & 4 & 1 \\ 2 & -3 & 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -2 & 5 \\ 0 & 0 & 9 \\ 3 & 0 & 4 \end{bmatrix}$

(c) The standard matrix for the transformation defined by $T(x_1, x_2) = (4x_1 - 2x_2, 6x_1 - 3x_2)$

(d) The matrix A given that AB is invertible.

(e) The matrix A where $A\mathbf{x} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$ does not have a solution.

8. Find A^{-1} for

$$(a) A = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$$

$$(b) A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

9. Chapter 1 - Supplementary Exercises - pages 102 & 103 -# 1, 5, 6

10. Chapter 2 - Supplementary Exercises - pages 183 & 184 -# 1, 8, 9