

## Review Questions for Exam 2

1. Find a basis for the set of all vectors of the form  $\begin{bmatrix} a - 2b + 5c \\ 2a + 5b - 8c \\ -a - 4b + 7c \\ 3a + b + c \end{bmatrix}$  where  $a, b, c \in \mathbb{R}$ .

2. Consider the polynomials  $\mathbf{p}_1 = 1 + t$ ,  $\mathbf{p}_2 = 1 - t$ ,  $\mathbf{p}_3 = 4$ ,  $\mathbf{p}_4 = t + t^2$ ,  $\mathbf{p}_5 = 1 + 2t + t^2$ . Let  $H$  be the subspace spanned by the set  $S = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5\}$ . Find a basis for  $H$ .

3. Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be a linear transformation where  $T(\mathbf{x}) = A\mathbf{x}$  for  $A = \begin{bmatrix} 1 & -5 & 3 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -2 \end{bmatrix}$ .

- (a) What is the dimension of the range of  $T$ ? Find a basis for the range of  $T$ .
- (b) What is the dimension of the kernel of  $T$ ? Find a basis for the kernel of  $T$ .

4. Find the characteristic polynomial and the eigenvalues for the matrix  $A = \begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -2 \\ 0 & -4 & 3 \end{bmatrix}$ .

5. In a kindergarten class, each day a child may choose to color with one of two crayons. Each day 50% of the students class who chose a blue crayon the day before choose a blue crayon again and 40% of the students who chose a red crayon the day before choose a red crayon again.

- (a) What is the stochastic matrix for this situation?
- (b) If on the first day 35% of the students choose a red crayon, what percentage of the students have a red crayon at the end of the second day?
- (c) What percentage of the students have a blue crayon at the end of the year?

6. Find a basis for the eigenspace corresponding to  $\lambda = -2$  for  $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$ .
7. Let  $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$  and  $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$  bases for a vector space  $V$ . Let  $\mathbf{b}_1 = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$ ,  $\mathbf{b}_2 = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$ ,  $\mathbf{c}_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ , and  $\mathbf{c}_2 = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ .
- (a) Find  $P_{\mathcal{C} \leftarrow \mathcal{B}}$  and  $P_{\mathcal{B} \leftarrow \mathcal{C}}$ .
- (b) If  $[\mathbf{x}]_{\mathcal{C}} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ , find the coordinate vector of  $\mathbf{x}$  in  $\mathcal{B}$ .
8. Determine if the set  $H = \left\{ \begin{bmatrix} 0 & a \\ 0 & a+b \end{bmatrix} : a, b \in \mathbb{R} \right\}$  is a subspace of  $M_{2 \times 2}$ . If  $H$  is a subspace, what is its dimension?
9. Find a basis for Row  $A$  for the matrix  $A$  in problem 3.
10. Be able to determine if the statements in problem 1 in the Supplementary Exercise at the end of Chapter 4 (pages 298-299) are True or False.