MA 522 - Homework 4:

Due Wed. Sep. 30, 2009

Problem 1. 2.17

Problem 2. (PROGRAMMING) Write a MATLAB M-file to solve $Ax = b$ by

1. Gaussian elimination with partial pivoting
2. by first computing A^{-1} (using matlab `inv(A)`) and then multiplying $x = inv(A) * b$.

You should write your own M-file to compute LU factorization of A by Gaussian elimination with partial pivoting, but do use `inv(A)` for A^{-1}.

Run your program for the following two sets of linear systems and print the condition number of the matrix, the relative error $\|\hat{x} - x\|_\infty / \|x\|_\infty$, the relative residual $\|r\|_\infty / (\|A\|_\infty \|\hat{x}\|_\infty)$ in a table form, where $r = b - A\hat{x}$.

1. The $n \times n$ Hilbert matrix $A = [1/(i + j - 1)]$ and $b = [1, 1, \cdots, 1]^T$. Test your codes for $n = 5, 10, 20, 50$.

   note: The inverse of Hilbert matrix has integer entries and can be generated using matlab's `invhilb(n)`. Then $x = A^{-1}b$ can be computed exactly (all operations are integer operations). The Hilbert matrix itself can be generated using matlab's `hilb(n)`.

2. The matrix is a random $n \times n$ matrix with condition number c as generated by the code `genmat(n, c)`, which can be downloaded from the class web page. For this test, generate a random vector $\mathtt{x} = \mathtt{rand}(\mathtt{n}, 1)$ and form $b = A * x$. Use $n = 20$ and $c = 1e1, 1e4, 1e8, 1e12, 1e16$.

Discuss your results.