MA 114 — Calculus II Spring 2011
Exam 1

This is a closed book exam. There are nine (9) problems on ten (10) pages (including this cover page). Check and be sure that you have a complete exam.

No books or notes may be used during the exam. You may use a graphing calculator provided that it does not have symbolic manipulation capabilities. In addition, any device capable of electronic communication (cell phone, pager, etc.) must be turned off and out of sight during the exam.

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. If you need more space then use the backs of the exam pages.

Show your work. Answers without justification will receive no credit. Partial credit for a problem will be given only when there is coherent written evidence that you have solved part of the problem. In particular, answers that are obtained simply as the output of calculator routines will receive no credit. Finally, be aware that it is not the responsibility of the grader to determine which part of your response is to be graded. Be sure to erase or mark out any work that you do not want graded.

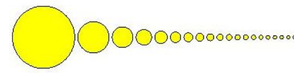
Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Problem	Score	Total
1		12
2		12
3		12
4		11
5		12
6		11
7		10
8		10
9		10
		100

1) (12 pts) In the diagram there are an infinite number of circles, $\{C_n\}_{n=1}^{\infty}$. Circle C_1 is the largest, C_2 the next largest, etc. For each n, the radius of C_n is $\frac{1}{n^{\frac{3}{4}}}$ cm.



(a) Write a formula for A_n, the area (in cm^2) of the n^{th} circle. Show your work.

(b) Write a formula for P_n, the circumference (in cm) of the n^{th} circle. Show your work.

(c) Determine whether the total area of all of the circles is finite. That is, determine whether the series $\sum_{n=1}^{\infty} A_n$ is convergent or not. Explain your answer.

(d) Determine whether the total perimeter of all of the circles is finite. That is, determine whether the series $\sum_{n=1}^{\infty} P_n$ is convergent or not. Explain your answer.

2) (12 pts) Let $f(x) = \sum_{n=1}^{\infty} n^4 \frac{(2x)^n}{3^n}$.

(a) Calculate the **interval of convergence** of f. Be sure to check end points.

(b) Calculate a power series representation for $f'(x)$ (i.e. express $f'(x)$ in the form $f'(x) = \sum_{n=1}^{\infty} a_n x^n$). Show your work.

(c) What is the **radius of convergence** of $f'(x)$? Explain your answer.

3) (12 pts)

Complete the following.

(a) If $S = \sum_{n=1}^{\infty} a_n$ is a series and N is a positive integer then S_N, the N^{th} **partial sum** of S, is defined to be

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(b) If $\sum_{n=1}^{\infty} a_n$ is a series and L is a number then $\sum_{n=1}^{\infty} a_n = L$ means precisely

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(c) The sequence $\{a_n\}_{n=1}^{\infty}$ is **bounded below** means that there is a number M such that

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(d) If $S = \sum_{n=1}^{\infty} c_n (x-a)^n$ is a power series then the **interval of convergence of S** is defined to be

____

4) (11 pts)

All parts of this problem refer to the telescoping series $S = \sum_{k=1}^{\infty} a_k$

where $a_k = \dfrac{1}{\ln(k+1)} - \dfrac{1}{\ln(k+2)}$.

(a) Express the partial sum $S_N = \sum_{k=1}^{n} a_k$ in terms of N.

(b) Use your answer to part (a) and the definition of series convergence to show that the series $\sum_{k=1}^{\infty} a_k$ converges.

(c) Calculate the exact value of $\sum_{k=1}^{\infty} a_k$.

5) (12 pts)

(a) State the **Alternating Series Test** in full.

(b) Carefully apply the alternating series test to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{e^n}$ to show that the series converges. You must verify all of the hypothesis.

(c) Apply the **Alternating Series Estimation Theorem** to the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{e^n}$ and determine the least *integer* N such that the N^{th} partial sum S_N is within .01 of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{e^n}$. Show your work.

6) (11 pts) You must carefully explain your answers to each of the following.

(a) For which values of s is the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{s^2-24}}$ divergent?

(b) What are the values of s for which $\displaystyle\sum_{n=1}^{\infty} (2s+1)^n$ is convergent?

7) (10 pts) Analyze each of the following series for convergence. You must justify your answers.

(a) Does $\sum_{n=1}^{\infty} \frac{n\sqrt{n}}{(n+1)(n+2)}$ converge or diverge? Use one or more convergence tests to justify your answer. State the name of each convergence test that you use and explain carefully how you apply it.

(b) Does $\sum_{n=1}^{\infty} \frac{n!}{(-2)^n}$ converge or diverge. Use one or more convergence tests to justify your answer. State the name of each convergence test that you use and explain carefully how you apply it.

8) (10 pts) Determine whether each of the following converges or diverges. You must justify your answers.

(a) Does $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4+3\,n+7}}{\sqrt{n}\,(n^3+2)}$ converge or diverge? Use one or more convergence tests to justify your answer. State the name of each convergence test that you use and explain carefully how you apply it.

(b) Does $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n+1}{\sqrt{2\,n^2+10}}\right)^n$ converge or diverge? Use one or more convergence tests to justify your answer. State the name of each convergence test that you use and explain carefully how you apply it.

9) (10 pts) **You must justify your answers to the following.**

(a) Consider the sequence of numbers $\{\frac{1}{n}\}_{n=1}^{\infty}$.

1. Give an upper bound for the above sequence.

2. Give a lower bound for the above sequence, and

3. Give a number which is neither an upper nor a lower bound for the above .

(b) Give an example of a divergent series $\sum_{i=i}^{\infty} a_i$ for which $\sum_{i=i}^{\infty} a_i^2$ is convergent.

(c) Give an example of a series which fails to converge by the **Test for Divergence**.