This is a closed book exam. No books or notes are to be used during the exam. You may use a graphing calculator if it does not have symbolic manipulation capabilities. However, any device capable of electronic communication (cell phone, pager, etc.) must be turned off and out of sight during the exam.

Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. Partial credit for a problem will be given only when there is coherent written evidence that you have solved part of the problem. In particular, answers that are obtained simply as the output of calculator routines will receive no credit. *Show your work.* Answers without justification will receive no credit.

Name: ______________________________

Section: __________

Last four digits of student identification number: __________

Problem	Score	Total
1		23
2		16
3		4
4		11
5		20
6		8
7		8
8		9
9		10
10		11
Total		120

1) (23 pts) Compute the following integrals:

(a) (7 pts) $\int x\sin(5x)dx.$ $= -\frac{x\cos(5x)}{5} + \int \frac{\cos(5x)}{5}\,dx$

$\begin{pmatrix} u = x, & dv = \sin(5x)dx \\ du = dx, & v = -\frac{\cos(5x)}{5} \end{pmatrix}$ $= -\frac{x\cos(5x)}{5} + \frac{\sin(5x)}{25} + C$

(b) (7 pts) $\int x\sqrt{4-x^2}dx.$ $= -\frac{1}{2}\int u^{\frac{1}{2}}du = -\frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$

$u = 4-x^2$ $= -\frac{1}{3}(4-x^2)^{3/2} + C$

$du = -2x\,dx$

(c) (9 pts) $\int_0^1 \sqrt{4-x^2}dx.$ $= \int_0^{\frac{\pi}{6}} 2\cos(\theta)\cdot 2\cos(\theta)\,d\theta$

$x = 2\sin(\theta)$ $= 2\int_0^{\frac{\pi}{6}} 2\cos^2(\theta)\,d\theta$

$dx = 2\cos(\theta)d\theta$ $= 2\int_0^{\frac{\pi}{6}} (1+\cos(2\theta))\,d\theta$

$x=0 \Rightarrow \theta = 0$

$x=1 \Rightarrow \theta = \frac{\pi}{6}$ $= 2\left(\theta + \frac{\sin(2\theta)}{2}\right)\Big|_0^{\frac{\pi}{6}}$

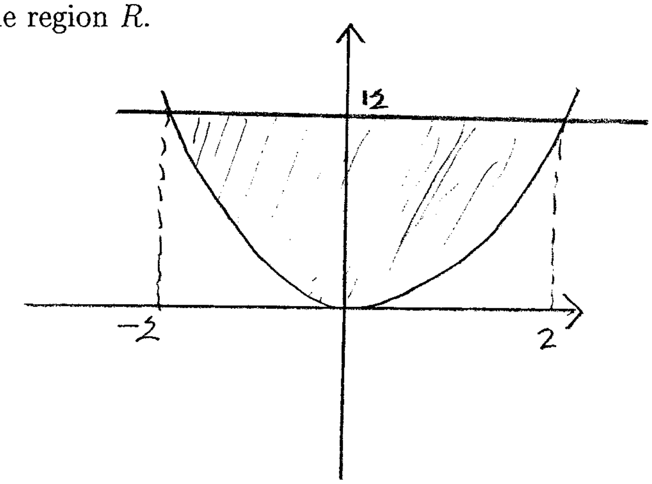
$= 2\left(\frac{\pi}{6} + \frac{\sin(\frac{\pi}{3})}{2}\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$

2) (16 pts) Let R be the name of the region enclosed by the curves $y = 3x^2$ and $y = 12$.

(a) (3 pts) Sketch and shade in the region R.

$3x^2 = 12$

$\Rightarrow x = \pm 2$



(b) (6 pts) Compute the area of the region R.

$$A = 2\int_0^2 (12 - 3x^2)\,dx = 2(12x - x^3)\Big|_0^2$$
$$= 2(24 - 8) = 32$$

(c) (7 pts) Set up an integral that computes the volume of the solid that is obtained by rotating the region R about the x-axis. State whether you are using the method of washers, the method of cylindrical shells or another method.

Washers

$$V = \pi \int_0^2 \left[12^2 - (3x^2)^2 \right] dx$$

3) (4 pts) Obtain the exact value of $\int_4^6 \left(\frac{1}{x} + \frac{1}{3-x}\right) dx$ in terms of logarithms. Simplify your answer as much as possible.

$$= \left[\ln|x| - \ln|3-x|\right]\Big|_4^6$$

$$= [\ln 6 - \ln 3] - [\ln 4 - \cancel{\ln 1}]$$

$$= \ln 6 - \ln 12 = \ln \frac{6}{12} = \ln \frac{1}{2} = -\ln 2$$

4) (11 pts) Let C be the curve defined parametrically by $x = e^{2t}$, $y = e^{3t}$, $0 \le t \le 1$.

(a) (4 pts) Eliminate the parameter t to find the Cartesian equation of C.

$$x = e^{2t} \Rightarrow x^3 = e^{6t}$$

$$y = e^{3t} \Rightarrow y^2 = e^{6t}$$

So $x^3 = y^2$

(b) (7 pts) Find the length of C.

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(2e^{2t})^2 + (3e^{3t})^2}\, dt$$

$$= \int_0^1 \sqrt{4e^{4t} + 9e^{6t}}\, dt$$

$$= \int_0^1 \sqrt{e^{4t}(4 + 9e^{2t})}\, dt$$

$$= \int_0^1 e^{2t}(4 + 9e^{2t})^{\frac{1}{2}}\, dt \qquad u = 4 + 9e^{2t}, \quad du = 18e^{2t}\, dt$$

$$= \frac{1}{18}\int_{13}^{4+9e^2} u^{\frac{1}{2}}\, du = \frac{1}{18}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big|_{13}^{4+9e^2}$$

$$= \frac{1}{27}\left[(4+9e^2)^{3/2} - 13^{3/2}\right]$$

5) (a) (6 pts) Find the sum or show the divergence of $\sum_{n=0}^{\infty}(-1)^n \frac{3^n}{5^{n+1}}$.

$$\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{5^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{5}\left(-\frac{3}{5}\right)^n = \sum_{n=0}^{\infty} a r^n = \frac{a}{1-r}, \ |r|<1$$

$r = -\frac{3}{5}$ and $|r|<1$

So $\frac{a}{1-r} = \frac{1/5}{1-(-3/5)} = \frac{1/5}{1+3/5} = \frac{1}{8}$

(b) (6 pts) Determine whether the series $\sum_{k=1}^{\infty}(-1)^k \frac{\sin(k)}{k^3+1}$ is convergent. Give the name of any of the tests you apply and show how you apply them.

Show that it is absolutely convergent (AC) and use AC $\Rightarrow$ $\boxed{\text{convergence}}$

$$\sum \left| (-1)^k \frac{\sin(k)}{k^3+1} \right| = \sum \frac{|\sin(k)|}{k^3+1}$$

But $0 \leq \frac{|\sin(k)|}{k^3+1} \leq \frac{1}{k^3}$ and

$\sum \frac{1}{k^3}$ is convergent by p-series test $(p=3>1$

So $\sum \frac{|\sin(k)|}{k^3+1}$ is convergent by comparison test

(Problem 5 continued on next page)

(c) (8 pts) Determine the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{2^n}$. Remember to consider the endpoints.

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(n+1)(x+3)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n(x+3)^n} \right|$$

$$= \frac{n+1}{n} \cdot \frac{1}{2} \, |x+3| \xrightarrow{n\to\infty} \frac{1}{2}|x+3| < 1$$

or $|x+3| < 2$

$-2 < x+3 < 2$

$-5 < x < -1$

$x=-5$ $\quad \sum \frac{n(-2)^n}{2^n} = \sum (-1)^n n$ Diverges since $\lim (-1)^n n \neq 0$

$x=-1$ $\quad \sum \frac{n(2)^n}{2^n} = \sum n$ also Diverges

So interval is $(-5,-1)$

6) (8 pts) Find the Maclaurin series of $f(x) = \frac{1}{(x+5)^2}$ and determine its radius of convergence. Express your answer using sigma notation.

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \quad , \quad R = 1$$

$$\frac{5}{5} \cdot \frac{1}{1+\frac{x}{5}} = \sum (-1)^n \left(\frac{x}{5}\right)^n = \sum \frac{(-1)^n}{5^n} x^n, \quad R = 5$$

$$\frac{5}{x+5} = \sum \frac{(-1)^n}{5^n} x^n$$

Diff.

$$-\frac{5}{(x+5)^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{5^n} \cdot n x^{n-1} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{5^{n+1}} (n+1) x^n$$

$$\frac{1}{(x+5)^2} = \sum_{n=0}^{\infty} \left(-\frac{1}{5}\right) \frac{(-1)^{n+1}}{5^{n+1}} (n+1) x^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{5^{n+2}} (n+1) x^n$$

$$R = 5$$

7) (8 pts) Solve the initial-value problem $y' = 2x(y^2+1)$, $y(0) = 1$. Express your answer explicitly as a function of x.

$$\frac{dy}{y^2+1} = 2x \, dx$$

$$\tan^{-1}(y) = x^2 + c$$

$$y = \tan(x^2 + c)$$

$$1 = \tan(c) \Rightarrow c = \frac{\pi}{4}$$

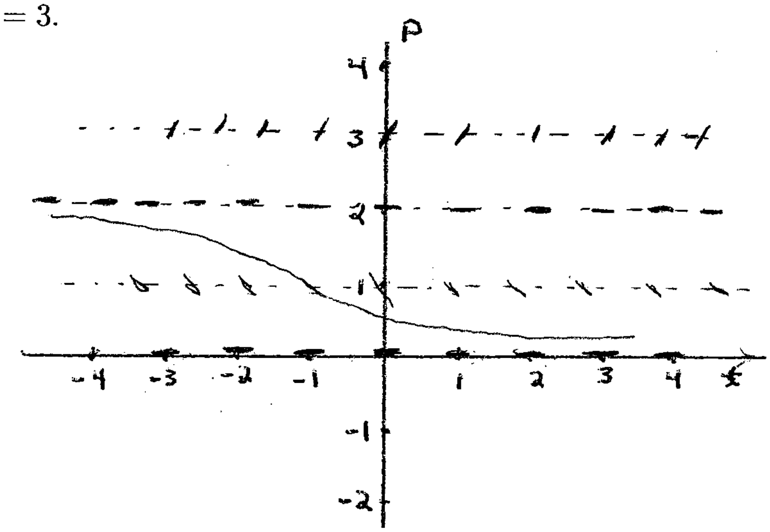
$$y = \tan\left(x^2 + \frac{\pi}{4}\right)$$

8) (9 pts) This problem considers the differential equation $\frac{dP}{dt} = P(P-2)$.

(a) (2 pts) Find all the equilibrium solutions.

$P = 0,\ P = 2$

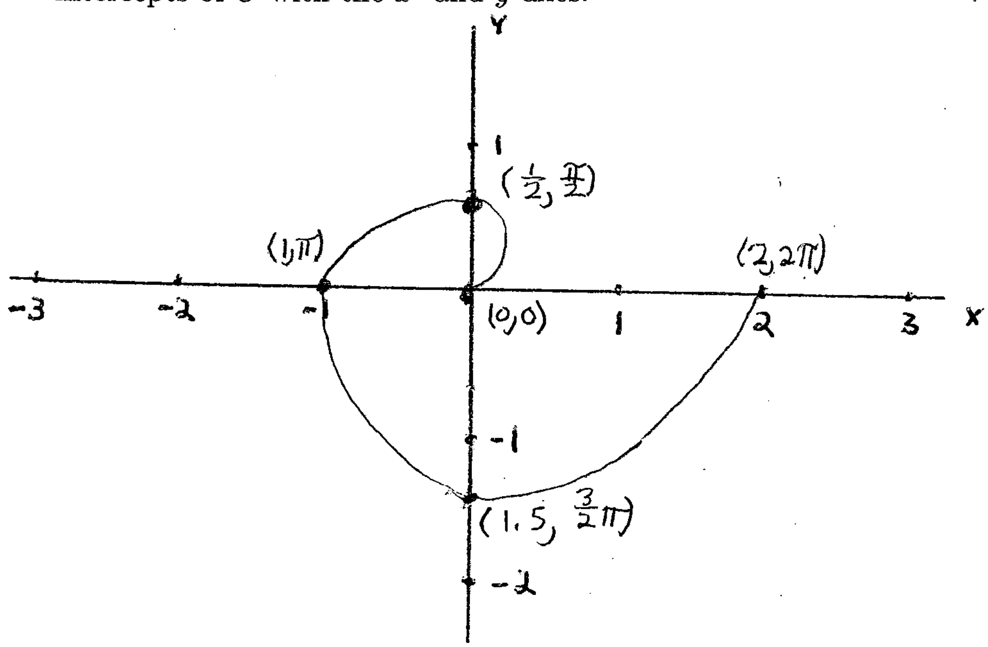
(b) (5 pts) Sketch the portion of the direction field on the lines $P = 0$, $P = 1$, $P = 2$ and $P = 3$.



(c) (2 points) Plot the solution curve with $P(0) = .7$ on the graph for part (b).

9) (10 pts) Let C be the polar curve $r = \theta/\pi,\ 0 \le \theta \le 2\pi$.

(a) (5 pts) Sketch the graph of C. Include on your graph the polar coordinates of the intercepts of C with the x- and y-axes.



(Problem 9 continued on next page)

(b) (5 pts) Find the slope of the line tangent to the graph of C at the point where $\theta = \frac{\pi}{2}$.

$$\theta = \frac{\pi}{2} \Rightarrow (x,y) = (0, \tfrac{1}{2})$$

$$\frac{dy}{dx} = \frac{(r\sin(\theta))'}{(r\cos(\theta))'} = \frac{(\frac{\theta}{\pi}\sin(\theta))'}{(\frac{\theta}{\pi}\cos(\theta))'} = \frac{1\cdot\sin(\theta) + \theta\cos(\theta)}{1\cdot\cos(\theta) + \theta(-\sin(\theta))}$$

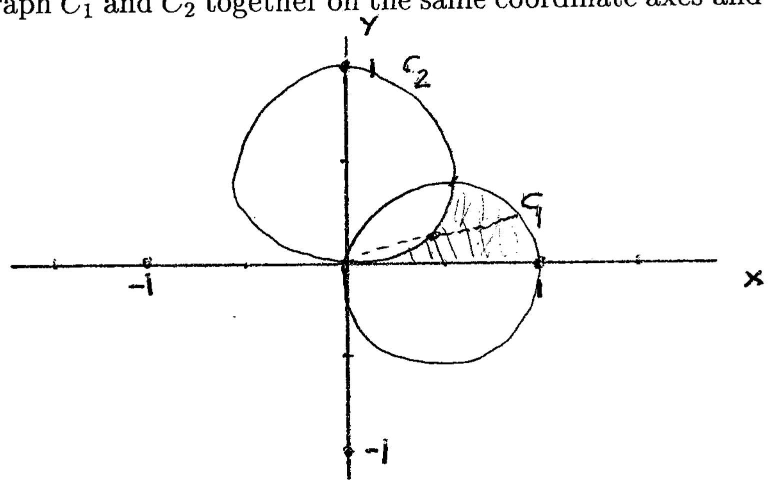
$$= \frac{1+0}{0-\frac{\pi}{2}} = -\frac{2}{\pi}$$

$$m = -\frac{2}{\pi},\ (x_0, y_0) = (0, \tfrac{1}{2})$$

$$y = \tfrac{1}{2} + (-\tfrac{2}{\pi})(x-0)$$

10) (11 pts) Let C_1 be the polar curve $r = \cos(\theta)$ and let C_2 be the polar curve $r = \sin(\theta)$.

(a) (6 pts) Graph C_1 and C_2 together on the same coordinate axes and identify each curve.



(b) (5 pts) Set up an integral using polar coordinates that computes the area of the region in the first quadrant that lies inside of C_1 and outside of C_2. Shade in the area you are computing on the graph in part (b).

$$\cos(\theta) = \sin(\theta) \Rightarrow \theta = \frac{\pi}{4}$$

$$A = \frac{1}{2}\int_0^{\frac{\pi}{4}} (r_1^2 - r_2^2)\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} (\sin^2\theta - \cos^2\theta)\,d\theta$$