Worksheet 7. MA113
Calculus I Fall 2006

As always, your work should be written out neatly and carefully. Use complete sentences.

1. Let $f(x) = \int_0^x \frac{1}{2+t^2}\,dt$. Find the interval(s) where f is increasing and the interval(s) where f is decreasing. Find the interval(s) where f is concave up and the interval(s) where f is concave down.

2. Suppose that f is a continuous function defined on the real line. Let $g(t) = \int_0^t f(s)\,ds$.

   (a) Compute $g'(t)$.

   (b) If $n > 0$, find an expression for $\frac{d}{dt}[(g(t))^{n+1}]$ and show that

   $$\int_0^x f(t)\left(\int_0^t f(s)\,ds\right)^n dt = \frac{1}{n+1}\left(\int_0^x f(s)\,ds\right)^{n+1}.$$

   Hint: Do not use the principle of mathematical induction. If F and G have the same derivative, what do we know about $F - G$?

3. The Great Pyramid at Giza is of height 135 meters and the base is a square with sides of length 230 meters. Follow the steps below to compute the volume in cubic meters of the Great Pyramid of Giza. You may check your answer using the formula $V = \frac{1}{3}Ah$ for the volume of a pyramid of height h and whose base has area A.

   (a) We view the pyramid as built up in layers starting with the base. Each layer is square slab with thickness Δy. Draw a picture of a pyramid and indicate a typical layer. Find an approximate value for the volume of a layer that is y meters above the ground and of thickness Δy. Give your answer in terms of y and Δy.

   (b) Pick a partition $0 = y_0 < y_1 < y_2 \ldots < y_n = 135$ which divides the pyramid into n slabs of equal thickness. Compute the volume of each slab and sum to obtain an approximation to the total volume of the pyramid.

   (c) As the thickness of each layer tends to zero, the sum approaches a definite integral. Find this integral and evaluate it to give the volume of the pyramid.