Worksheet on the definition of a derivative. MA113
Calculus I Fall 2006

This worksheet is designed to provide practice in using the definition of the derivative to find the slope of a tangent line. Some of the calculations are sometimes a bit tedious. However, they provide good practice in algebra. More importantly, we should learn the basics (nothing in calculus is more basic than the definition of the derivative) well, before we move on to more elegant approaches.
As always, your work should be written out neatly and carefully. Use complete sentences.
For these exercises, it will be useful to recall that we may remove radicals from the denominator of an expression by multiplying by the conjugate:

$$\frac{1}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}.$$

1. Write the definition of the derivative of a function f at a point x.

2. Use the definition of the derivative to find the derivative of $f(x) = \sqrt{2-3x}$. What is the domain of f? What is the domain of f'?

3. Suppose (x, y) is a point on a circle centered at the origin. Using well-forgotten facts from geometry, find the slope of the tangent line to the circle which passes through (x, y). State clearly the fact from geometry that you use to find the slope.

4. Now consider the unit circle, the circle of radius one which is centered at the origin. The part of the unit circle that lies above the x-axis is give by the graph of the function $g(x) = \sqrt{1-x^2}$. Use the definition of the derivative to find the slope of the tangent line to the unit circle at the point $(x, \sqrt{1-x^2})$.

5. Do your answers to parts 3 and 4 agree?

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