

Answer the following questions. Display your answers clearly and neatly. Explain your reasoning. Use complete sentences.

1. In this problem we try to approximate the sine function on the interval $[0, \pi]$ by a quadratic function which has the same zeroes as the sine function on this interval. There are many quadratic functions which have zeroes at 0 and π . In each part, we impose an additional condition which guarantees that there is only one function.
 - (a) Find a quadratic function f such that $f(0) = f(\pi) = 0$ and which has the same maximum value as the sine function on the interval $[0, \pi]$.
 - (b) Find a quadratic function g such that $g(0) = g(\pi) = 0$ and which has the same tangent line at 0 and π as the sine function.
 - (c) Find a quadratic function h such that $h(0) = h(\pi) = 0$ and so that the area above the x -axis and below the graph of h is the same as for the sine function.
2. We can view the sum $\sum_{k=1}^{10,000} \sqrt{k}$ as a Riemann sum for an integral. Evaluate the integral to obtain an approximate value for the sum. Is your approximate value smaller or larger than the correct value of the sum? Hint: Use a partition where each subinterval is of width 1.
3. If $x \sin(\pi x) = \int_0^{2x} f(t) dt$ and f is continuous, find $f(2)$.

(These questions are taken from Stewart, page 373.)

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