1 Lecture 8: The derivative as a function.

1.1 Outline

• Definition of the derivative as a function. definitions of differentiability.
• Differentiability implies continuity.
• Example: Finding a derivative.
• Example: Finding tangent lines.
• Examples: Points where a function is not differentiable.

1.2 The derivative

Definition. Given a function \( f \), we may define a new function \( f' \), which we call the derivative of \( f \) by

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h},
\]

provided the limit exists.

An equivalent definition that is sometimes useful is

\[
f'(x) = \lim_{y \to x} \frac{f(y) - f(x)}{y - x}.
\]

A function is differentiable at \( x \), if \( f'(x) \) exists. Thus the domain of \( f' \) is the set of values \( x \) so that \( f \) is differentiable at \( x \).

A function is differentiable on an interval \( I \) if \( f \) is differentiable for each \( x \) in \( I \).

1.3 Differentiability and continuity.

Theorem 1 If \( f \) is differentiable at \( x \), then \( f \) is continuous at \( x \).

1.4 Examples

Example. Find the derivative of \( f(x) = \sqrt{x} \).

Example. Let \( f(x) = 1/x \). Find all values \( x \) where \( f'(x) = 4 \). Find all value \( x \) where \( f'(x) = -4 \).

Find all tangent lines to the graph of \( f \) which are parallel to the line \( y = -4x \).

Example. Let \( f(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases} \). Show that \( f \) is not differentiable at 0.
Example. Let $f(x) = |x|$. Where is $f$ continuous? Where is $f$ differentiable?

Example. Let $f(x) = \sqrt[3]{x}$. Give the domain. Show $f$ is not differentiable at 0.

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