1. Follow the steps below to find the tangent line to the function $f(x) = 1/x$ at $x = 2$.

   (a) The first step is compute the slope of the line that passes through the points $(x, f(x))$ and $(2, f(2))$.

   $$\frac{f(x) - f(2)}{x - 2}.$$

   Write out this expression for $f(x) = 1/x$.

   $$\frac{f(x) - f(2)}{x - 2} = \frac{1}{x - 2}\left(\frac{1}{x} - \frac{1}{2}\right).$$

   (b) The slope of the tangent line is

   $$\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}.$$

   Can we apply the rule for the limit of a quotient to find this limit?

   Not unless we have simplified. As written the limit of the denominator, $\lim_{x \to 2}(x - 2) = 0$.

   Can we use direct substitution to find the limit?

   No. The expression $\frac{f(x) - f(2)}{x - 2}$ is undefined at $x = 2$.

   If these rules do not apply, does the mean the limit does not exist?

   No. As we will see below, we can simplify and obtain a limit that is easy to evaluate.

   (c) Simplify the expression

   $$\frac{f(x) - f(2)}{x - 2}.$$

   $$\begin{aligned} \frac{f(x) - f(2)}{x - 2} &= \frac{1}{(x - 2)}\left(\frac{1}{x} - \frac{1}{2}\right) \\ &= \frac{1}{(x - 2)}\left(\frac{2}{2x} - \frac{x}{2x}\right) \\ &= \frac{1}{(x - 2)}\frac{2 - x}{2x} \\ &= \frac{-1}{2x} \end{aligned}$$

(d) Find the slope of the tangent line.

Using our answer from the previous part,

$$\begin{aligned}\lim_{x\to 2}\frac{f(x)-f(2)}{x-2} &= \lim_{x\to 2}\frac{-1}{2x}\\ &= \frac{-1}{4}\end{aligned}$$

After simplifying the function $-1/(2x)$ is continuous at $x = 2$ and we can use substitution or the rule for the limit of a quotient to evaluate the limit.

(e) Find the equation of the tangent line to the graph of $y = 1/x$ at $x = 2$.

From the above the tangent line has slope $-1/4$ and it must pass through the point $(2, f(2))$. Thus the equation of the line is

$$(y - \frac{1}{2}) = \frac{-1}{4}(x - 2).$$

This simplifies to

$$y = \frac{-1}{4}x + 1.$$

We may check by graphing.