

1. Compute the derivative of

$$f(x) = (x^3 + 2x)e^x.$$

Using the product rule

$$\begin{aligned} f'(x) &= (x^3 + 2x)'e^x + (x^3 + 2x)(e^x)' \\ &= (3x^2 + 2)e^x + (x^3 + 2x)e^x \\ &= (x^3 + 3x^2 + 2x + 2)e^x. \end{aligned}$$

2. Find the equation of the line tangent to the graph of the function

$$f(x) = \frac{2-x}{1+x}$$

at the point $(1, f(1))$.

- (a) Compute the derivative $f'(x)$ and compute $f'(1)$.

From the quotient rule we have

$$\begin{aligned} f'(x) &= \frac{(-1)(1+x) - (2-x)1}{(1+x)^2} \\ &= \frac{-3}{(1+x)^2}. \end{aligned}$$

Evaluating at $x = 1$ gives that $f'(1) = -3/4$.

- (b) Write out the equation of the tangent line and check by graphing f and the tangent line.

The tangent line passes through the point $(1, f(1)) = (1, 1/2)$ and the equation is $y - \frac{1}{4} = -\frac{3}{4} \cdot (x - 1)$. Simplifying gives the equation

$$y = -\frac{3}{4}x + \frac{5}{4}.$$