*Calculus II* *Fall 2003*
*MA114:007-009* *Russell Brown*

The final exam will take place 1-3pm on Wednesday, 17 December 2003 in CB 122. Approximately half of the exam will cover new material, §§10.9, 10.10, 10.12, 9.1–9.4 and approximately half of the exam will consist of questions from the first three exams. I will make small changes in the questions which do not affect the method of solution.
The new topics to be examined are:

1. Differentiating and integrating power series.
2. Finding the series for $\ln(1+x)$ and $\tan^{-1}(x)$.
3. Taylor and McLaurin series.
4. Finding series for $\sin x$, $\cos x$ and $e^x$.
5. Use of Taylor's theorem to estimate the error when approximating a function by a Taylor polynonial.
6. Use of the alternating series estimation theorem to estimate the error when approximating a series by a Taylor polynomial.
7. Parametric curves. Rewriting as Cartesian equations.
8. Tangent lines to parametric curves.
9. Arclength of parametric curves.
10. Polar coordinates. Sketching curves in polar coordinates.

Review assignment

- Chapter 10 Review, pp 675–677. #61, 63, 65, 66, 67, 71.
- §10.10 #37, 38, 40.
- Chapter 9 Review, pp. 591–592. #1–4, 5–19, 17–23, 33, 34.

Sample exam questions

1. Compute 4 of the 5 indefinite integrals below. Write here ______ the letter of the integral that is not to be graded. If you do not specify the an integral which is not to be graded, we will take the four lowest scores.

   (a) $\int \frac{x}{x^2+4}\,dx$

(b) $\displaystyle\int \frac{1}{(9-x^2)^{3/2}}\,dx$

(c) $\displaystyle\int \frac{x}{x^2+6x+10}\,dx$

(d) $\displaystyle\int x\sin(3x)\,dx$

(e) $\displaystyle\int \frac{1}{2+\sqrt{x}}\,dx$

2. Find the values of $\lambda$ for which $y(x) = e^{\lambda x}$ satisfies the equation

$$y'' - 4y' - 5y = 0.$$

3. Find the sum of each of the following series.

(a) $\displaystyle\frac{1}{4\cdot 3} + \frac{1}{4\cdot 9} + \frac{1}{4\cdot 27} + \frac{1}{4\cdot 81} + \ldots + \frac{1}{4\cdot 3^n} + \ldots$

(b) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{3}{n} - \frac{3}{n+1}\right)$

4. (a) Write the MacLaurin series or Taylor series about 0 for $e^x$.

(b) Use your answer in part a) to find the MacLaurin series for $e^{-x^3}$.

(c) Find the MacLaurin series for

$$\int_0^x e^{-t^3}\,dt.$$

5. The trapezoid rule $T_n$ and Simpson's rule $S_n$ for approximating the integral $\int_a^b f(x)\,dx$ are

$$\begin{aligned} T_n &= \frac{h}{2}(f(x_0) + 2f(x_1) + \ldots + 2f(x_{n-1}) + f(x_n)) \\ S_n &= \frac{h}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)) \end{aligned}$$

The errors satisfy

$$|E_T| \le \frac{M_2(b-a)^3}{12n^2} \quad \text{and} \quad |E_S| \le \frac{M_4(b-a)^5}{180n^4}$$

where $M_k$ is a number which satisfies $|f^{(k)}(x)| \le M_k$ for all $x$ with $a \le x \le b$.

(a) Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate the integral

$$\int_4^7 \cos(3x)\,dx.$$

Give your answers correctly rounded to four decimal places.

(b) Find $n$ so that the error in the trapezoid rule is at most $10^{-4}$,

$$\left|\int_4^7 \cos(3x)\,dx - T_n\right| \leq 10^{-4}.$$

(c) Find $n$ so that the error in Simpson's rule is at most $10^{-4}$,

$$\left|\int_4^7 \cos(3x)\,dx - S_n\right| \leq 10^{-4}.$$

6. (a) Sketch the parametric curve defined by $x(t) = 2t^2$, $y(t) = t^3 - t$, for $-2 < t < 2$.
   (b) Find the values of $t$ so that $(x, y) = (2, 0)$.
   (c) Find all tangent lines to this curve at $(x, y) = (2, 0)$.
7. (a) Write the length of the curve $x(t) = t^2$, and $y(t) = t^3$ for $0 \leq t \leq 1$ as an integral.
   (b) Evaluate the integral in part a).
8. (a) Sketch the curve defined in polar coordinates by $r = 2\cos(3\theta)$.
   (b) Give an interval of $\theta$ which corresponds to one leaf of this curve.

December 7, 2003