Calculus III  
MA213:007–008  
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Announcements.

- The second hour exam will be on Friday, 5 November 2004, 2:2:50 in CB 349.
- Please read the syllabus to find out if you may use your calculator on the exam.
- The final exam will be 1-3pm on Wednesday, 15 December 2004 in our regular (lecture) classroom, CB 349. Please visit http://www.uky.edu/Registrar/finals-fall.html for the University final exam schedule. The time listed in the course calendar is not correct.
- Below are several review questions and problems to help you prepare for the exam. In addition, you should study your notebook problems to help prepare for the exam.
- The exam will cover sections §§12.3–12.8 and §§13.1-13.4, 13.6. See the assignment sheets for a list of topics. See your notes from lecture for a more detailed description of the material covered.

Review questions and problems.

1. Be familiar with the following bits of notation.

\[ f_x, \frac{\partial f}{\partial x}, D_u f, \nabla f, =, \int \int \]

2. What does it mean for a function \( f(x, y) \) to be differentiable at \( (x_0, y_0) \)?

State a theorem which can be used to show that a function is differentiable.

3. Give the geometric interpretation of the gradient.

4. Let \( f(x, y) = \cos(x^2 + y^2) \). For which unit vectors \( u \) does the directional derivative of \( f \), \( D_u f(\sqrt{\pi}, 0) = 0 \)?

5. Find a point on the surface \( x^2 + y^2 - 2z^2 = 23 \) and find the tangent plane to the surface at that point.

6. If \( f_x(2, 3) = 1, f_y(2, 3) = 2, x(4, 5) = 2, y(4, 5) = 3, x_u(4, 5) = 6, x_v(4, 5) = 7, y_u(4, 5) = 8 \) and \( y_v(4, 5) = 9 \), then find \( f_u(x(4, 5), y(4, 5)) \).

Do you need to know if \( x = x(u, v) \) or \( x = x(v, u) \)?

7. Find the differential of the function for the volume of a right circular cone \( V = \frac{1}{3} \pi r^2 h \).

If \( V(r_0, h_0) = 10 \), can you estimate \( V(r_0 + 0.01, h_0 + 0.02) \)?

8. Find two unit tangent vectors to the surface \( z = \cos(x) \cos(y) \) at the point where \( x = \pi/6 \) and \( y = \pi/4 \).

9. Let \( f \) be a differentiable function of one variable. If \( u(x, t) = f(x + 2t) \), show that \( u_{tt} - 4u_{xx} = 0 \).
10. Give a geometric interpretation of the Lagrange multiplier condition for constrained local extreme values.

11. Give a geometric interpretation of the integral
\[ \iint_R f(x, y) \, dxdy. \]

12. Be able to define regions of type I and type II.

13. State Fubini’s theorem.

14. Find the value of the integral
\[ \int_0^1 \int_x^1 \sin(y^2) \, dy \, dx. \]

15. Consider the surface \( S = \{(x, y, z) : z = f(x, y), (x, y) \in R\} \) which is a graph over a region \( R \) in the \((x, y)\)-plane. Compare the area of the surface \( S \) and the area of the region \( R \). Which is larger?

16. The following questions from homework would be good examination problems.
   \[ \S 12.3 \#50, 76a, \S 12.4 \#3, 33, \S 12.5 \#17, 33, \S 12.6 \#19, 43, \S 12.7 \#30, 45, \S 12.8 \#15, 23. \]
   \[ \S 13.2 \#23, \S 13.3 \#15, 25, 39, \S 13.4 \#9, 21. \S 13.6 \#1, 7. \]

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