Homework 5. Due Wednesday, 19 October 2005.

1. Evans, p. 85 #5. Hint: Find a constant $M$ so that $\pm u(x) + M(1 - |x|^2)$ is subharmonic. Use the maximum principle.

2. Evans, p. 85 #8. For simplicity, you may assume that $g(x) = |x|$ if $|x| \leq 1$ and $g(x) = 0$ for $|x| > 1$. However, as Evans indicates, as long as $g$ is bounded, the conclusion is independent of the values of $g$ outside a neighborhood of the origin.

   This problem is interesting because it shows that if $g$ is Lipschitz, we do not necessarily have that $u$ is Lipschitz. There is a class called the Zygmund class which is slightly larger than the class of Lipschitz functions and so that if $g$ is in the Zygmund class, then $u$ is in the Zygmund class. This helps to explain why we have a sawtooth metal strip on every package of Saran wrap.\(^1\)

3. Suppose that $A$ is a linear transformation and $\Delta(u \circ A) = (\Delta u) \circ A$ for all $C^2$ functions $u$. Show that $A$ is orthogonal. Hint: The functions $x_i x_j$ might be useful.

\(^1\)I have heard that this last observation is due to Zygmund.