MA575
MWF 9-9:50pm
CB 343
Fall 2006

Due Wednesday, 13 September 2006.

1. (Beals, page 20) For two sets $A$ and $B$, we define
   \[ A + B = \{ c : c = a + b \text{ with } a \in A, \ b \in B \} \]. Show that $\sup A + B = \sup A + \sup B$.

2. Can you show that if $E$ is a set of real numbers, then $\sup E \in E$?

3. (Beals, page 20) Show that, for any positive reals $x$ and $y$, there is a positive integer $n$ so that $x/n < y$.

4. (Beals, page 20) The nested interval property. Suppose that $I_1, I_2, \ldots$ is a sequence of bounded closed intervals with $I_1 \subset I_2 \subset I_3 \ldots$. Let $I_n = [a_n, b_n]$ and suppose that $\lim_{n \to \infty} b_n - a_n = 0$.
   
   Show that $\bigcap_{n=1}^{\infty} I_n$ is not empty. Is the result still true if the intervals are not bounded?
   
   Hint: What can you say about $\sup \{a_n : n = 1, 2, \ldots \}$.

5. (Beals, page 20) Prove that for any positive $h$ and any integer $n \geq 0$, we have
   \[(1 + h)^n \geq 1 + nh + \frac{n(n-1)}{2} h^2.\]
   
   Hint: Binomial theorem. You may assume the binomial theorem.

6. (Beals, page 20) Suppose that $a$ is positive and $n \geq 2$ is an integer. Suppose that $a^n = n$. Prove that $1 < a < 1 + \sqrt{2/(n-1)}$.

7. Prove that $\lim_{n \to \infty} n^{1/n} = 1$.

8. Negate the definition of a sequence which converges to $a$. That is write out what it means for a sequence $\{a_n\}$ to not converge to $a$.

9. (Beals, page 29) Prove that there is no way to put an order relation on $C$ so that $C$ is an ordered field. An ordered field satisfies the axioms A1-4, M1-4, D and O1-4.

10. (Beals, page 29) Show that if $a$, $b$ and $c$ are complex numbers, $|a| = |b| = |c|$ and $a + b + c = 0$, then $|a - b| = |b - c| = |a - c|$. Discuss the geometric significance.

Additional questions.(Not to be collected.)

1. Is $\pi^e$ transcendental?