Math 114
Spring 2009
Prof. Readdy

1. Estimate $\int_1^2 \sqrt{x}dx$ with
   (a) 4 trapezoids using the trapezoidal rule;
   (b) 4 rectangles using the midpoint rule.
   (c) What is the smallest number of rectangles needed using the midpoint rule to obtain an estimate with error $< .1$?

2. (a) Use the Trapezoidal rule and Simpson's rule to estimate from the data below the value of the integral $\int_0^6 f(x)dx$.

x	0	1	2	3	4	5	6
$f(x)$	1	2	5	5	3	2	1

   (b) It is known that $-1 \leq f''(x) \leq 2$ and $-3 \leq f^{(4)}(x) \leq 1$ for all $x \in [0,6]$.
      i. Estimate the error involved in each of the approximations in part (a).
      ii. What is the smallest n for which we know for sure that the approximation S_n from Simpson's rule to the integral $\int_0^6 f(x)dx$ is accurate to within 10^{-3}?

3. Show that $\int_0^1 \frac{1}{x^p}\,dx$ converges for $0 < p < 1$ and diverges for $p \geq 1$.

4. Determine whether each integral below is convergent or divergent. Evaluate those that are convergent.
   (a) $\int_{-\infty}^{\infty} x\,dx$
   (b) $\int_1^9 \frac{1}{(x-9)^{1/3}}\,dx$
   (c) $\int_1^4 \frac{1}{x^2+x-6}\,dx$

5. Solve the given differential equations.
   (a) $\frac{dy}{dx} = \frac{y^2}{x}$;
   (b) $\frac{dy}{dx} = e^{x+y}$;
   (c) $\frac{dy}{dx} = xy + 4y + 3x + 12$.

6. Solve the following initial value problems:

   (a) $\dfrac{dy}{dx} = \dfrac{y^2 - 1}{x}, \qquad y(1) = 2;$

   (b) $xy\dfrac{dy}{dx} = \ln x, \qquad y(1) = 2.$

7. Three grams of a radioisotope decay in two years to 0.9 g. Determine both the half-life T and the decay constant k.

8. Suppose that a sum A_0 is invested at an annual rate of return r compounded continuously. Find the return rate that must be achieved if the initial investment is to double in 8 years.

9. Find the arc length of the curve $y = mx + b$ for $0 \le x \le 10$ in two ways: geometrically and using Calculus. Here m and b are arbitrary constants.

10. Find the arc length of the curve $y = 2\ln(\sin(x/2))$ for $\pi/3 \le x \le \pi$

11. Find the arc length of the curve $x = \frac{2}{3}(y-1)^{3/2}$ for $1 \le y \le 4$.

12. Find the arc length of the curve $x = \frac{y^2}{2}$ for $0 \le x \le 1/2$. You should assume y is positive.

13. Draw the slope field for $y' = x + y$ for $-2 \le x, y \le 2$. Sketch the graphs of the solutions that satisfy the initial conditions: (a) $y(0) = 0$, (b) $y(0) = 1$ (c) $y(0) = -.1$.

14. (a) Use Euler's method with step size $h = .1$ to estimate the value of $y(.4)$, where y is the solution of the initial value problem $y' = y$, $y(0) = 1$.

    (b) Solve the initial value problem. Check your answer is correct.

15. Chapter 9 review (16) on world population.

16. Give the first four terms of the sequence $\{\frac{n}{n+1}\}_{n\ge 0}$.

17. Find a formula for the general term of the sequence $1, \frac{1}{2}, 3, \frac{1}{4}, 5, \frac{1}{6}, \ldots$.

****

Helpful Equations you will be given for the exam:

If $|f''(x)| \leq K$ for all $a \leq x \leq b$, then

$$|Error(\text{Trap}_n)| \leq K\frac{(b-a)^3}{12n^2}$$

and

$$|Error(\text{Midpoint}_n)| \leq K\frac{(b-a)^3}{24n^2}$$

If $|f^{(4)}(x)| \leq C$ for all $a \leq x \leq b$, then

$$|Error(\text{Simpson}_n)| \leq C\frac{(b-a)^5}{180n^4}$$

REMINDER:

EXAM II
WEDNESDAY, MARCH 11, 2009
TIME: 9:00 AM TO 9:50 AM
LOCATION: CB 102

MATERIAL:
SECTIONS 7.7, 7.8, 8.1, 9.1, 9.2, 9.3, 9.4, 11.1 (A LITTLE)

PLEASE REMEMBER TO BRING PHOTO ID TO THE EXAM.
NO CALCULATORS ARE ALLOWED FOR THE EXAM.