

Exam III Review Questions

1. Consider the series  $\sum_{i=1}^{+\infty} a_i$ .
  - (a) Define the  $n$ th partial sum  $s_n$ .
  - (b) Give the definition for a series to be convergent. When is a series divergent?
2. For each of the following series, determine if it converges absolutely, converges conditionally or diverges.

(a)  $\sum_{n \geq 0} \frac{2^n}{5^n}$

(b)  $\sum_{n \geq 0} \frac{(-1)^n}{5^n}$

(c)  $\sum_{n \geq 0} \frac{3n + 1}{5n - 2}$

(d)  $\sum_{n \geq 1} \frac{1}{\sqrt{n}}$

(e)  $\sum_{n \geq 1} \frac{1}{n}$

(f)  $\sum_{n \geq 1} \frac{\sin n}{n^2}$

(g)  $\sum_{n \geq 1} \frac{\sin n\pi}{n}$

(h)  $\sum_{n \geq 1} \frac{(-1)^n}{n}$

(i)  $\sum_{n \geq 1} \frac{5n^2 + 3n - 17}{n^{18} - 5n^2 + 19}$

(j)  $\sum_{n \geq 1} \left(1 + \frac{1}{n}\right)^2 e^{-n}$

(k)  $\sum_{n \geq 1} \frac{1}{n!}$

(l)  $\sum_{n \geq 1} \frac{n!}{n^n}$

3. Find the power series for

(a)  $\frac{1}{1+x^3}$

(b)  $\frac{1}{(1-x)^3}$

4. Show that each of the following series is convergent and find their sum:

(a)  $\sum_{n \geq 1} \frac{1}{n(n+1)}$

(b)  $\sum_{n \geq 2} \frac{(-2)^n}{7^n}$

(c)  $6 + 4 + \frac{8}{3} + \frac{16}{9} + \frac{32}{27} + \dots$

5. Determine whether each of the following series converges

(a)  $\sum_{n \geq 2} \frac{1}{n \ln n}$

(b)  $\sum_{n \geq 1} \frac{1}{n^7}$

(c)  $\sum_{n \geq 1} \frac{1}{n^{-7}}$

(d)  $\sum_{n \geq 1} \frac{1}{3^n + n}$

(e)  $\sum_{n \geq 1} \frac{n^2 + 2}{n^4 + 5}$

(f)  $\sum_{n \geq 1} \frac{(-1)^n n^2}{n^2 + 5}$

(g)  $\sum_{n \geq 1} \frac{1}{n^2 - 1}$

6. Show that the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

converges, and find how many terms you would need to add to find its sum accurate to within 0.01.

7. Determine if the series converges or diverges. If it converges find its sum.

$$\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right).$$

8. Consider the series

$$\sum_{n=0}^{\infty} \frac{1}{2^{n+(-1)^n}} = \frac{1}{2} + \frac{1}{1} + \frac{1}{8} + \frac{1}{4} + \cdots$$

- (a) Use the root test to decide if the series converges or diverges.
- (b) Use the fact that a certain geometric series is absolutely convergent to conclude if the series converges or diverges.

9. Express the integral

$$\int_0^1 \frac{1 - \cos(x)}{x^2} dx$$

as an alternating series. What is the smallest  $n$  for which the partial sum  $s_n$  approximates the integral to five decimal places? Approximate the integral to five decimal places.

10. Be sure to review sequences!

REMINDER:

EXAM III

WEDNESDAY, APRIL 15, 2009

TIME: 9:00 AM TO 9:50 AM

LOCATION: CB 102, OUR USUAL CLASSROOM

MATERIAL:

SECTIONS 11.1 THROUGH 11.8

PLEASE REMEMBER TO BRING PHOTO ID TO THE EXAM.

NO CALCULATORS ARE ALLOWED FOR THE EXAM.