## Math 714 Spring 2020

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# POLYTOPES: A MODERN VIEW

#### Abstract

This will be a topics course on polytopes and the algebraic framework and techniques developed in the past 15 years to study their facial incidence structure. The course will give the core results in the theory of coalgebras applied to polytopes, and include new research directions which embrace quasisymmetric functions, topology, algebraic geometry, Kazhdan-Lusztig theory, and polynomial flag vector inequalities. This course will have something for everyone.

The first third of this course is based upon notes for the lecture series "Polytopes" I gave at the Institute for Advanced Study in 2013. Since then there have been some exciting developments related to understanding Kazhdan-Lusztig polynomials, Bruhat graphs and more general manifolds.

No previous knowledge of polytopes or combinatorics is assumed. Prerequisite: One year of a prelim sequence course or permission of instructor.

## 1. Introduction

Posets, Polytopes, Coalgebras.

#### 2. Flag enumeration

Face vectors, Eulerian posets, The cd-index.

#### 3. Coalgebra techniques

The coalgebras of posets and **ab**-polynomials, Geometric operations on polytopes, Spanning results.

#### 4. Whitney stratified spaces and Euler enumeration

Quasi-graded posets, a generalization of the Eulerian condition, Whitney stratifications, Merging strata, Extending the semi-suspension operation, The shelling components.

#### 5. Shellings and linear inequalities

Bruggesser-Mani line shellings, Spherical shellings, The lifting technique, Minimization of the **cd**-index for polytopes.

#### 6. The cd-index of balanced graphs

Labeled acyclic digraphs, The balanced condition, The  $\tilde{r}$  and  $\tilde{f}$  polynomials, Bruhat graphs of Coxeter groups as a special case.

### 7. Other topics (time permitting)

Kazhdan-Lusztig polynomials and the **cd**-index, Oriented matroids and flag enumeration, Manifold arrangements, Karu's interpretation of the **cd**-index coefficients, Minimization of the **cd**-index for Gorenstein\* lattices.