

MATH 714 FALL 2011

PROF. MARGARET READDY

## POLYTOPES: THE ABCD'S OF ENUMERATING FLAGS

(AKA POLYTOPES: ABCDEF)

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### Course Description

This is a topics course on polytopes and the algebraic framework and techniques developed in the past 15 years to study their facial incidence structure. The course will give the core results in the theory of coalgebras applied to polytopes, and include new research directions which embrace quasisymmetric functions, topology, algebraic geometry, Kazhdan-Lusztig theory, polynomial flag vector inequalities and Whitney stratified spaces. This course will have something for everyone.

This course is based upon the research monograph I am writing with Richard Ehrenborg and Louis Billera entitled, "Polytopes: The ABCD's of Enumerating Flags".

No previous knowledge of polytopes or combinatorics is assumed.

### Textbook

None. I will be putting a number of items on reserve and will provide a reading list as the course progresses.

### Homework and Grading

Approximately every two weeks we will hold a homework session during classtime in which we discuss the homework problems. You can discuss the homework with your classmates or with me. You are not allowed to discuss the homework with anyone else, including other graduate students and faculty.

As this is a topics course, you are expected to demonstrate to me that you are making an effort to learn the course material. Ways you can do this include: attending lecture regularly, asking or answering questions in class, presenting solutions during the homework sessions, giving a talk on a paper in the Discrete CATS seminar. If you wish to give a Masters talk, please let me know early enough so that we can select a paper.

## Schedule of Topics

### 1. Introduction

Posets, Polytopes, Coalgebras.

### 2. Flag enumeration

Face vectors, Eulerian posets, The **cd**-index.

### 3. Coalgebra techniques

The coalgebras of posets and **ab**-polynomials, Geometric operations on polytopes, Spanning results, The toric  $g$ -vector.

### 4. Generalized Dehn-Sommerville relations

The coalgebra of flag operators, The ideal of the generalized Dehn-Sommerville relations,  $k$ -Eulerian posets.

### 5. Zonotopes and poset transforms

Hyperplane arrangements and zonotopes, Oriented matroids, Zaslavsky's and Bayer-Sturmfels' results, The **c**, **2d**-index of a zonotope, The signed Birkhoff transform, The Tchebyshev transform.

### 6. Shellings and linear inequalities

Bruggesser-Mani line shellings, Spherical shellings, The lifting technique, Minimization of the **cd**-index for polytopes.

### 7. Algebraic geometry and polytopes

Toric varieties, Techniques for Gorenstein\* lattices, Karu's interpretation of the **cd**-index coefficients, Minimization of the **cd**-index for Gorenstein\* lattices.

### 8. Quasisymmetric functions

The quasisymmetric function of a poset, The Stembridge peak algebra.

### 9. New horizons

Kazhdan-Lusztig polynomials, The Charney-Davis conjecture, Polynomial inequalities for the flag  $f$ -vector, Results for other manifolds, Whitney stratified spaces.