"Algebraic and Analytic Combinatorics"
October 28-29, 2006
AMS Fall Eastern Meeting
University of Connecticut
Storrs, CT
Organized by Richard Ehrenborg and Margaret A. Readdy

Saturday, October 28th
Morning session

8:00 am	**Richard Stanley, MIT** **"A Conjectured Combinatorial Interpretation of the Normalized Irreducible Character Values of the Symmetric Group"**
8:30 am	**Lauren Williams, Harvard** **"A Markov chain on permutation-tableaux whose quotient is the asymmetric exclusion process"**
9:00 am	**Robin Pemantle, University of Pennsylvania** **"Generating functions: converting algebraic information into asymptotic estimates"**
9:30 am	**Denis Chebikin, MIT** **"Counting permutations by 3-descents"**
10:00 am	**Gábor Hetyei, University of North Carolina at Charlotte** **"Tchebyshev transforms and the roots of the derivative polynomials for tangent and secant"**
10:30 am	**Alex Postnikov, MIT** **"Combinatorics of Grassmann cells"**

Saturday, October 28th
Afternoon session

2:30 pm	**Louis Billera, Cornell** **"The complete cd-index of a Bruhat interval and a simple expression for its Kazhdan-Lusztig polynomial"**
3:00 pm	**Alex Iosevich, University of Missouri** **"Sum/Product theorems in finite fields via Kloosterman sums"**

3:30 pm **Isabella Novik, University of Washington**
"How many edges can a centrally symmetric polytope have?"

4:00 pm **Peter Perry, University of Kentucky**
"Counting Pattern-Avoiding Permutations with Perron and Frobenius"

4:30 pm **Sergi Elizalde, Dartmouth**
"A bijection between 2-triangulations and pairs of non-crossing Dyck paths"

5:00 pm **Matthias Beck, San Francisco State**
"Cyclotomic Polytopes and Growth Series of Cyclotomic Lattices"

Sunday, October 29th
Morning session

8:00 am **Gil Kalai, Yale and Hebrew University**
"Harmonic analysis of Boolean functions"

8:30 am **Sinai Robins, Temple**
"Polyhedral theta functions"

9:00 am **Michael Slone, University of Kentucky**
"An index for non-regular spheres"

9:30 am **Pavlo Pylyavskyy, MIT**
"Non-Crossing Tableaux"

10:00 am **Patricia Hersh, Indiana University**
"Coloring complexes and arrangements"

10:30 am **Ira Gessel, Brandeis**
"Is Analysis Necessary?"

Sunday, October 29th
Afternoon session

2:30 pm **Jakob Jonsson, Royal Institute of Technology and MIT**
"On the Torsion Part of the Homology of the Matching Complex"

3:00 pm **Kevin Costello, UCSD and Rutgers**
"The Rank of Random Graphs"

ABSTRACTS

Matthias Beck* and Serkan Hosten, **San Francisco State University**, **"Cyclotomic Polytopes and Growth Series of Cyclotomic Lattices"**.
The coordination sequence of a lattice L encodes the word-length function with respect to M, a set that generates L as a monoid. We investigate the coordination sequence of the cyclotomic lattice $L = \mathbf{Z}[\zeta_m]$, where ζ_m is a primitive m'th root of unity and where M is the set of all m'th roots of unity. We prove several conjectures by Parker regarding the structure of the rational generating function of the coordination sequence; this structure depends on the prime factorization of m. Our methods are based on unimodular triangulations of the m'th cyclotomic polytope, the convex hull of the m roots of unity in $\mathbf{R}^{\phi}(m)$, and combine results from commutative algebra, number theory, and discrete geometry.

Louis Billera*, **Cornell**, **"The complete cd-index of a Bruhat interval and a simple expression for its Kazhdan-Lusztig polynomial"**.
We define a nonhomogeneous extension of the usual **cd**- index of a Bruhat interval in an arbitrary Coxeter group, called the *complete* **cd**-index. This is shown to have many of the algebraic properties of the usual (homogeneous) **cd**-index of the underlying Eulerian poset. In the case of a Bruhat interval that is a lattice, for example, the complete **cd**-index reduces to the usual one.

We give an expression for the Kazhdan-Lusztig polynomial of a Bruhat interval in terms of its complete **cd**-index. Other than this **cd**-index, this expression involves only simple - explicitly defined - polynomials. As one corollary, we obtain a formula of Bayer and Ehrenborg that gives the intersection homology Betti numbers of a toric variety in terms of the usual **cd**-index of the underlying polytope.

Denis Chebikin*, **MIT**, **"Counting permutations by 3-descents"**.
Given a permutation $\sigma \in S_n$, we say that σ has a 3-*descent* at position i if the sequence of 3 consecutive elements of σ starting at position i forms an odd permutation. In this paper we consider the problem of counting permutations by the number of 3-descents, and more generally, by the set of positions at which a permutation has a 3-descent. We relate the number of permutations with a fixed 3-descent set to the number of permutations with a certain ordinary descent set. Then we derive a generating function enumerating permutations by length and number of 3-descents and discuss combinatorial properties of noncommutative polynomials enumerating permutations of given length by 3-descents. This is joint work with Pavlo Pylyavskyy.

Kevin Costello*, **UCSD and Rutgers**, **"The Rank of Random Graphs"**.
We consider the adjacency matrix of a random graph, and in particular the following two questions:

1. Is the matrix almost surely (non-)singular?
2. If the matrix is singular, how close is it likely to be to full rank?

We will discuss answers to the two questions for the Erdös-Rényi graph $G(n, p)$ with edge probability in the range $\frac{\ln n}{2n} < p < \frac{1}{2}$, along with some conjectured answers for the case of random regular graphs.

Sergi Elizalde*, **Dartmouth**, **"A bijection between 2-triangulations and pairs of non-crossing Dyck paths"**.
Triangulations of a convex polygon are known to be counted by the Catalan numbers. A natural generalization of a triangulation is a k-triangulation, which is defined to be a maximal set of diagonals so that no $k+1$ of them mutually cross in their interiors. It was proved by Jonsson that k-triangulations are enumerated by certain determinants of Catalan numbers, that are also known to count k-tuples of non-crossing Dyck paths.

There are several simple bijections between triangulations of a convex n-gon and Dyck paths. However, no bijective proof of Jonsson's result is known for general k. Here we solve this problem for $k = 2$, that is, we present a bijection between 2-triangulations of a convex n-gon and pairs (P, Q) of Dyck paths of semilength $n-4$ so that P never goes below Q. The bijection is obtained by constructing isomorphic generating trees for the sets of 2-triangulations and pairs of non-crossing Dyck paths.

Ira Gessel*, **Brandeis**, **"Is Analysis Necessary?"**.
Analytic techniques are often used to prove results, such as combinatorial identities, whose statement involves no analysis. I will discuss the question of whether analysis is really necessary to prove such results, with examples involving integration, residues, P- recursive sequences, and asymptotic series.

Patricia Hersh*, Indiana University-Bloomington, **and Edward Swartz, Cornell University**, **"Coloring complexes and arrangements"**.
We provide a convex ear decomposition for the coloring complex of any finite graph and for some related simplicial complexes. As a consequence, we deduce new constraints on the chromatic polynomials of all finite graphs, by using a formula of Steingrimsson which relates the chromatic polynomial of a finite graph to the h-polynomial of the double cone of its coloring complex.

Gábor Hetyei*, **University of North Carolina at Charlotte**, **"Tchebyshev transforms and the roots of the derivative polynomials for tangent and secant"**.
Inspired by the recently discovered relation between the Tchebyshev polynomials and the derivative polynomials for secant that arises in the study of Tchebyshev transforms of posets, we show that the roots of the derivative polynomials for tangent and secant are all distinct, pure imaginary, located between $-i$ and i, and interlaced. The proof uses combinatorics and analytic results related to the argument principle. We will also discuss some generalizations of our statement and the techniques used in its proof.

Alex Iosevich*, **University of Missouri**, **"Sum/Product theorems in finite fields via Kloosterman sums"**.
Let $A \subset \mathbb{Z}_q$, the cyclic group with q elements, q prime. Bourgain, Katz and Tao proved that if $|A| \le Cq^{1-\epsilon}$, $\epsilon > 0$, there exists $\delta > 0$ such that $\max\{|A \cdot A|, |A + A|\} \ge C'|A|^{1+\delta}$.. We prove an "effective" version of this theorem which, for example, implies that if $|A| \approx q^{15/22}$, then $\max\{|A \cdot A|, |A + A|\} \ge C'|A|^{16/22}$. Our main tools are additive combinatorics and sharp bounds for Kloosterman type sums.

Jakob Jonsson*, Royal Institute of Technology and MIT, "On the Torsion Part of the Homology of the Matching Complex".
The matching complex M_n is the simplicial complex of matchings in the complete graph K_n. We present several results about the torsion part of the homology of M_n. First, there is nonvanishing 3-torsion in $\tilde{H}_d(M_n;\mathbb{Z})$ whenever $\nu_n \le d \le \lfloor \frac{n-6}{2} \rfloor$, where $\nu_n = \lceil \frac{n-4}{3} \rceil$. By results due to Bouc and to Shareshian and Wachs, $\tilde{H}_{\nu_n}(M_n;\mathbb{Z})$ is a nontrivial elementary 3-group for almost all n and the bottom nonvanishing homology group of M_n for all $n \neq 2$. Second, $\tilde{H}_d(M_n;\mathbb{Z})$ is a nontrivial 3-group whenever $\nu_n \le d \le \lfloor \frac{2n-9}{5} \rfloor$. Third, for each $k \ge 0$, there is a polynomial $f_k(r)$ of degree $3k$ such that the dimension of $\tilde{H}_{k-1+r}(M_{2k+1+3r};\mathbb{Z}_3)$, viewed as a vector space over $\mathbb{Z}_3$, is at most $f_k(r)$ for all $r \ge 0$. Fourth, using a result due to Anderson, we detect 5-torsion in the bottom nonvanishing homology of M_{14}. In addition, we present a few results with a similar flavor about the chessboard complex.

Gil Kalai*, Yale and Hebrew University, "Harmonic analysis of Boolean functions".
Boolean functions, namely functions $f(x_1, x_2, \ldots, x_n)$ are those where each variable as well as the value of the function itself attain the value 0 or 1. Boolean functions are fundamental objects in combinatorics, complexity theory, probability and other areas. Fourier analysis of Boolean functions has played an important role in these areas since the mid-eighties. Fourier analysis is related to discrete isoperimetric results, threshold phenomena for probabilistic models such as random graphs and percolations, low complexity classes, hardness of approximation and noise sensitivity. Hypercontractive estimates, namely, results asserting that certain operators contract even when considered from 2-norm to p-norm for $p > 2$ play (rather mysteriously) a crucial role. The talk, which will be self-contained, will discuss some of the developments in this area in a friendly way.

Isabella Novik*, University of Washington, and Alexander Barvinok, University of Michigan, "How many edges can a centrally symmetric polytope have?".
While the Upper Bound Theorem that provides sharp upper bounds on the face numbers of all polytopes is a classic by now, the situation for centrally symmetric polytopes is wide open. For instance, the largest number of edges, $e(d,n)$, that a d-dimensional centrally symmetric polytope on n vertices can have is unknown even for $d = 4$. We show that for (even) $d > 3$,

$$1 - 1/(d-1) + o(1) \le e(d,n)/\binom{n}{2} \le 1 - 1/2^d + o(1).$$

We also provide certain bounds on the maximal possible number of higher-dimensional faces. The methods we use come from elementary analysis.

Robin Pemantle*, University of Pennsylvania, "Generating functions: converting algebraic information into asymptotic estimates".
For univariate generating functions, f, there is a standard collection of techniques for converting information about f near its singularities to asymptotic information about its coefficients. For a large class of multivariate generating functions there is now also technology to do this. I will discuss the scope of this program.

Peter Perry*, **University of Kentucky**, **"Counting Pattern-Avoiding Permutations with Perron and Frobenius"**.
This talk reports on joint work with Richard Ehrenborg and Sergey Kitaev, and gives a new method for counting consecutive pattern-avoiding permutations using the spectral theory of integral operators. Let $\mathfrak{S}_n$ denote the symmetric group on n symbols; a *pattern* of length k is a subset S of $\mathfrak{S}_k$. For $x = (x_1, \cdots, x_k) \in \mathbb{R}^k$ with $x_i \neq x_j$ for $i \neq j$, let $\Pi(x)$ denote the unique permutation in $\mathfrak{S}_k$ with $\pi_i < \pi_j$ if and only if $x_i < x_j$, $1 \leq i < j \leq k$. A permutation $\pi \in \mathfrak{S}_n$ avoids the consecutive pattern S if $\Pi(x_1, \cdots, \pi_{j+k-1} \notin S$ for any j with $1 \leq j \leq n-k+1$.

We express the probability that a randomly selected $\pi \in \mathfrak{S}_n$ avoids S in terms of a positivity-preserving integral operator T_S which serves as a kind of transfer operator for the counting problem. The spectral theory of T_S determines the large-n asymptotics of this probability through Krein and Rutman's extension of the Perron-Frobenius theorem of matrix theory. We compute asymptotics in several cases of interest and represent the exponential generating function for the counting function in terms of a renormalized determinant of $(I - T_S)$.

Alex Postnikov*, **MIT**, **"Combinatorics of Grassmann cells"**.
We discuss the combinatorics that came from the totally positive Grassmannian. Simply put, we solve the following problem. Suppose we know that all maximal minors of a $k \times n$-matrix are either strictly positive or equal to zero. What can we say about possible configurations of nonzero minors? These configurations are given by certain geometrical objects that we call nonnegative Grassmann cells. We show that these cells are in bijection with many interesting combinatorial objects: L-diagrams, decorated permutations, planar networks modulo some transformations, alternating chord diagrams, rook placements on a certain skew chessboard, regions of a certain hyperplane arrangement, matroids of special kind, elements in some intervals in the Bruhat order, degenerations of cyclic polytopes, and many others. We construct explicit bijections between these seemingly different objects. We also discuss the partial order on these cells, which extends the Bruhat order.

Pavlo Pylyavskyy*, **MIT**, **"Non-Crossing Tableaux"**.
In combinatorics there is a well-known duality between non-nesting and non-crossing objects. In algebra there are many objects which are *standard*, for example Standard Young Tableaux, Standard Monomials, Standard Bitableaux. We adopt a point of view that these standard objects are really non-nesting, and we find their non-crossing counterparts.

Sinai Robins*, **Temple**, **"Polyhedral theta functions"**.
We develop the theory of polyhedral theta functions. Some (but not all) of these theta functions, defined over polyhedral cones, are in fact modular forms. Upon taking limits of these polyhedral theta functions in the modular variable, we obtain relations for solid angles of polytopes.

Michael Slone*, **University of Kentucky**, **"An index for non-regular spheres"**.
We study *weakly graded posets*, graded posets in which not all ranks must contain elements. We use these to define a modified ab-index, and show that an appropriately defined weakly graded Eulerian poset has a cd-index. Finally, we apply these results to the study of non-regular CW spheres.

Richard P. Stanley*, **MIT**, **"A Conjectured Combinatorial Interpretation of the Normalized Irreducible Character Values of the Symmetric Group"**.
Let χ^λ denote the irreducible (ordinary) character of the symmetric group $\mathfrak{S}_n$ indexed by the partition λ of n, and let $\chi^\lambda(\nu)$ denote the value of this character on a permutation of cycle type ν. Let μ be a partition of $k \leq n$, and let $(\mu, 1^{n-k})$ denote the partition of n obtained from μ by adjoining $n-k$ parts equal to 1. Regarding k as fixed, define the *normalized character*

$$\widehat{\chi}^\lambda(\mu, 1^{n-k}) = \frac{(n)_k \, \chi^\lambda(\mu, 1^{n-k})}{\chi^\lambda(1^n)},$$

where $\chi^\lambda(1^n)$ denotes the dimension of the character χ^λ and $(n)_k = n(n-1)\cdots(n-k+1)$. We discuss a conjectured combinatorial formula for $\widehat{\chi}^\lambda(\mu, 1^{n-k})$ generalizing the theorem (*Sem. Lotharingien de Combinatoire* (electronic) **50** (2003), B50d) when the shape of λ is a rectangle.

Lauren K. Williams*, **Harvard**, **and Sylvie Corteel, CNRS PRiSM Université de Versailles**, **"A Markov chain on permutation-tableaux whose quotient is the asymmetric exclusion process/ A Markov chain on permutations which projects to the PASEP"**.
The partially asymmetric exclusion process (PASEP) is an important model from statistical mechanics which describes a system of interacting particles hopping left and right on a one-dimensional lattice of N sites. It is partially asymmetric in the sense that the probability of hopping left is q times the probability of hopping right. Additionally, particles may enter from the left with probability α and exit from the right with probability β.

It has been observed that the (unique) stationary distribution of the PASEP has remarkable connections to combinatorics – see for example the papers of Derrida *et al*, Duchi and Schaeffer, Corteel, and Shapiro and Zeilberger. Most recently we proved that in fact the (normalized) probability of being in a particular state of the PASEP can be view ed as a certain weight generating function for *permutation tableaux* of a fixed shape. (This result implies the previous combinatorial results.) However, our proof relied on the *matrix ansatz* of Derrida *et al*, and hence did not give an intuitive explanation of why one should expect the steady state distribution of the PASEP to involve such nice combinatorics.

In this paper we define a Markov chain – which we call the PT chain – on the set of permutation tableaux which *projects* to the PASEP in a very strong sense. This gives a new proof of our previous result which bypasses the matrix ansatz altogether. Furthermore, via the bijection from permutation tableaux to permutations, the PT chain can also be viewed as a Markov chain on the symmetric group. Another nice feature of the PT chain is that it possesses a certain symmetry which extends the *particle-hole symmetry* of the PASEP.