

MA 162: Finite Mathematics

Fall 2014

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December 1, 2014

Announcements:

- First financial math homework due tomorrow at 6pm.
- Exam scores are posted. More about this on Wednesday.

Annuities and Loans

- Loans - Borrowing a large amount of money initially and paying it back in equal sized increments over time.
- Annuities - Investing equal amounts of money at regular intervals to obtain a certain amount of money at a future time.

Today we focus on loans.

A First Example

Jack borrows \$2000 today. He will repay the loan by making two equal payments over the next year. The payments will be made at the end of every six months. The interest is 4.1% APR compounded quarterly. Determine the size of Jack's payments.

A Problem with Our Current Method

Billy takes out a home loan worth \$175,000 today. He will repay the loan by making equal payments at the end of each month for the next 30 years. The interest is 5% APR compounded monthly. Determine the size of Billy's payments.

Computing Loan Payments Formula

- P denotes the principal of a loan (how much was borrowed)
- R denotes the size of the payment
- t denotes the number of years (the term of the loan)
- r is the nominal interest rate per year
- m is the number of conversion periods
- i is the interest rate per period, so $i = r/m$
- n is the number of conversion periods in the term, so $n = mt$
- Then

$$P = R \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Revisiting the Previous Examples

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Revisiting the Previous Examples

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Valuing a Loan at Different Times

Recall that Billy's home loan was for \$175,000 over 30 years compounded monthly at 5% APR. Exactly 5 years after Billy takes out this loan he wins the lottery. Billy would like to pay off his home loan at this point. How much money would this cost?

Things to Note in Billy's Example

- The total amount paid by Billy is

$$60(\$939.44) + \$160700.65 = \$217067.05$$

- He borrowed \$175000 so the total interest charge is

$$\$217067.05 - \$175000 = \$42067.05$$

- Had he continued to make regular payments for the full term of his loan, his total interest expense would have been \$163198.40.

- By paying off his loan early, Billy saved

$$\$163198.40 - \$42067.05 = \$121131.35$$

Refinancing Example

Kelsey takes out a home loan with \$250,000 principal. She makes payments at the end of each month for 30 years. The interest is 7.2% APR compounded monthly. Ten years into the loan, Kelsey considers refinancing her loan because interest rates have dropped to 6% APR compounded monthly. How much will Kelsey save on interest charges if she refinances?

More on Loan Payments

- Suppose you take out a loan of \$10000 at 5% APR compounded annually for a flexible amount of time.
- You are required to make a payment at the end of each year, but the amount is up to you.
- The loan will be settled as soon as the present value of all your payments equals the principal of the loan (present is considered at the beginning of the loan).
- What is the minimum payment you should make every year?

More on Loan Payments

What happens if you pay \$400 each year?

- At the end of the first year, the balance on the loan is $\$10000(1.05) = \10500 (this includes the interest).
- After applying your payment, this goes down to a balance of \$10100.
- After two years, the balance is $\$10100(1.05) = \10605 .
- This is reduced to \$10205 after your second payment.

More on Loan Payments

What happens if you pay \$500 each year?

- At the end of the first year, the balance on the loan is $\$10000(1.05) = \10500 (this includes the interest).
- After applying your payment, this goes down to a balance of \$10000.
- After two years, the balance is $\$10000(1.05) = \10500 .
- This is reduced to \$10000 after your second payment.

More on Loan Payments

What happens if you pay \$800 each year?

- At the end of the first year, the balance on the loan is $\$10000(1.05) = \10500 (this includes the interest).
- After applying your payment, this goes down to a balance of \$9700.
- After two years, the balance is $\$9700(1.05) = \10185 .
- This is reduced to \$9385 after your second payment.