

MA 162: Finite Mathematics

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Announcements:

- Second financial math homework due Friday at 6pm.
- Last financial math homework due next Friday (12/12) at 6pm.
- See the course webpage for final exam announcements - including an opportunity to increase your score on exam #3.

More on Loan Payments

- Suppose you take out a loan of \$10000 at 5% APR compounded annually for a flexible amount of time.
- You are required to make a payment at the end of each year, but the amount is up to you.
- The loan will be settled as soon as the present value of all your payments equals the principal of the loan (present is considered at the beginning of the loan).
- What is the minimum payment you should make every year?

More on Loan Payments

What happens if you pay \$400 each year?

- At the end of the first year, the balance on the loan is $\$10000(1.05) = \10500 (this includes the interest).
- After applying your payment, this goes down to a balance of \$10100.
- After two years, the balance is $\$10100(1.05) = \10605 .
- This is reduced to \$10205 after your second payment.

More on Loan Payments

What happens if you pay \$500 each year?

- At the end of the first year, the balance on the loan is $\$10000(1.05) = \10500 (this includes the interest).
- After applying your payment, this goes down to a balance of \$10000.
- After two years, the balance is $\$10000(1.05) = \10500 .
- This is reduced to \$10000 after your second payment.

More on Loan Payments

What happens if you pay \$800 each year?

- At the end of the first year, the balance on the loan is $\$10000(1.05) = \10500 (this includes the interest).
- After applying your payment, this goes down to a balance of \$9700.
- After two years, the balance is $\$9700(1.05) = \10185 .
- This is reduced to \$9385 after your second payment.

The Effect of Large Payments

- Suppose you take out a loan that you will repay by making 60 payments of \$100.
- Will making 30 payments of \$200 pay off the loan?
- Can we actually figure out how many payments it will take to pay off a loan by increasing the size of the payments?

Doubling Payments Example

Nick took out a car loan for \$10000. The loan is to be paid off with payments at the end of each month for 36 months at 2.7% APR compounded monthly. What is Nick's monthly payment?

Doubling Payments Example (cont.)

Nick took out a car loan for \$10000. The loan is to be paid off with payments at the end of each month for 36 months at 2.7% APR compounded monthly. Suppose Nick wants to double his car payment to pay the car off faster. How many payments will he need?

Doubling Payments Example (cont.)

How do we handle getting a fractional payment amount?

Annuities

An annuity is a sequence of payments made at regular time intervals. We will focus on ordinary simple level-payment annuity certainties. This means:

- the payments will be made at the end of each period
- the frequency of payments is the same as the frequency of interest compounding
- the cash flows are of equal sizes
- there are a fixed number of cash flows

Annuities

Here are several examples of annuities:

- You making regular payments into an account earning interest to reach a certain amount of money you want to have when you retire.
- A financial institution making regular payments to you from this retirement account for a certain number of years.
- Every loan can also be viewed as an annuity. For example, if you borrow \$20000 to buy a new car and pay it off in equally sized payments every month, then from the perspective of a car company they are being paid a certain amount of money every month. These both involve a steady stream of cash flows, the only difference is whether you view it as paying money or receiving money.

Annuity Example

Your dad just retired. His retirement plan will pay him \$1500 at the end of each month for the next 20 years out of an account that earns 4% compounded monthly. He would like to receive one lump sum right now so he can combine all his investments. How much money should his company pay him?

Future Value Annuity - Small Example

On December 1st you decide to deposit \$300 into a savings account that earns 3% APR compounded monthly at the end of every month for the next three months. How much money do you have in the account at the end of these three months?

Future Value of an Annuity

- F denotes the future value of the annuity (or loan)
- R denotes the payment size
- t denotes the number of years (the term of the annuity/loan)
- r is the nominal interest rate per year
- m is the number of conversion periods per year
- i is the interest rate period, so $i = r/m$
- n is the number of conversion periods in the term, so $n = mt$
- Then

$$F = R \left[\frac{(1 + i)^n - 1}{i} \right]$$

Computing the Future Value of an Annuity

Aaron recently decided to setup a retirement fund to plan for the future. He plans to deposit \$1700 into the account at the end of every 6 months until he retires 45 years from now. The retirement fund will earn 8% APR compounded semi-annually. How much money will be in the account when Aaron retires?