MA 162: Finite Mathematics

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Announcements:

- Last financial math homework due Friday at 6pm.
- See the course webpage for final exam announcements - including an opportunity to increase your score on exam #3.

Future Value of an Annuity

- F denotes the future value of the annuity (or loan)
- R denotes the payment size
- t denotes the number of years (the term of the annuity/loan)
- r is the nominal interest rate per year
- m is the number of conversion periods per year
- i is the interest rate period, so $i = r/m$
- n is the number of conversion periods in the term, so $n = mt$
- Then

$$F = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Computing the Future Value of an Annuity

Aaron recently decided to setup a retirement fund to plan for the future. He plans to deposit \$1700 into the account at the end of every 6 months until he retires 45 years from now. The retirement fund will earn 8% APR compounded semi-annually. How much money will be in the account when Aaron retires?

$R = \$1700$

$m = 2$

$t = 45$

$r = .08$

$i = \frac{r}{m} = \frac{.08}{2} = .04$

$n = mt = 2 \cdot 45 = 90$

$$F = 1700\left[\frac{(1.04)^{90} - 1}{.04}\right] = \boxed{\$1407571.67}$$

Relating Present and Future Values of Annuities

What is the present value of Aaron's annuity?

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right] = 1700\left[\frac{1-(1.04)^{-90}}{.04}\right]$$

$$= \boxed{\$41254.37}$$

What does this mean?

$$1407571.67 = 41254.37\left(1+\frac{.08}{2}\right)^{2\cdot 45}$$

The present is the amount Aaron would have to invest as one lump sum at the beginning of these 45 years to get the same future value.

Complex Word Problems

- The final exam will have approximately 25 points which come from complex word problems.
- Complex word problems will generally consist of multiple computations in order to get the final answer.
- You may have to use multiple formulas in a complex word problem as well.

Multiple Annuities

Helen plans to invest \$1000 at the end of every month for the next five years into an account which earns 4% APR compounded monthly. After these five years Helen plans to invest \$2500 at the end of every year for the next 30 years into an account which earns 3% APR compounded yearly. How much money does Helen have combined in the two accounts 35 years from now?

First 5 years

$$F = 1000\left[\frac{\left(1+\frac{.04}{12}\right)^{12\cdot 5}-1}{\left(\frac{.04}{12}\right)}\right] = 66298.98$$

→ move this from year 5 to year 35

From year 6 to year 35

$$F = 2500\left[\frac{(1+.03)^{30}-1}{.03}\right] = 118938.54$$

Total after 35 yrs is $66298.98\left(1+\frac{.04}{12}\right)^{12\cdot 30} + 118938.54$

$= \boxed{\$338620.08}$

Investing to receive regular payments

Ruth is planning ahead to finance a return to school. To pay for school, Ruth wants to invest money at the end of every 6 months for the next five years into a savings account that earns 8% APR compounded semiannually. Ruth expects to withdraw \$5000 semiannually during the following 4 years out the account to pay for her schooling. How much should she deposit every 6 months during the first five years?

Future value of the annuity formed by her 5 yrs of investing.

$$\underbrace{R\left[\frac{\left(1+\frac{.08}{2}\right)^{2.5}-1}{\left(\frac{.08}{2}\right)}\right]}_{\text{Future Value of all her payments five years from now.}} = \underbrace{5000\left[\frac{1-\left(1+\frac{.08}{2}\right)^{-8}}{\left(\frac{.08}{2}\right)}\right]}_{\text{Present value of all her withdrawals five years from now.}}$$

$$\boxed{R = \$2803.88}$$

Which loan is better?

You just bought a car for \$15000. The dealership gives you two loan options:

- 5 years, 4% APR compounded monthly
- 0% APR for the first year, and 6% APR compounded monthly for the remaining 4 years

Which loan requires you to pay less in interest charges (assume that you only make payments at the end of the months when interest is applied)?

Loan 1:

$$15000 = R\left[\frac{1-\left(1+\frac{.04}{12}\right)^{-12\cdot 5}}{\left(\frac{.04}{12}\right)}\right] \qquad R = 276.25$$

Interest Charges: $276.25(60) - 15000 = \underline{\$1575}$

Loan 2:

$$15000 = R\left[\frac{1-\left(1+\frac{.06}{12}\right)^{-48}}{\left(\frac{.06}{12}\right)}\right] \qquad R = 352.28$$

Interest Charges: $352.28(48) - 15000 = \underline{\$1909.44}$

Fourth Principle of Financial Mathematics

Assume the APR of an account earning compound interest is positive.

- The numerical value of the present value is not greater than the numerical value of the future value.

- To increase the difference between the present value and the future value, you can: (a) increase the APR, (b) increase the term, (c) increase the frequency of the payments.

- Let S be the sum of the payments; i.e. $S = nR$. Then the present value is not greater than the sum of the payments.

- Doubling the size of the regular payment will: (a) decrease the term of loan by more than $1/2$, (b) decrease the interest paid by more than $1/2$.

Formulas for the Exam

- Compound Interest

$$FV = PV(1+i)^n$$

- Compounded Continuously Interest

$$A = Pe^{rt}$$

- Present Value of Annuity/Loan

$$P = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

- Future Value of Annuity/Loan

$$F = R\left[\frac{(1+i)^n - 1}{i}\right]$$

- Effective Interest Rate

$$r_{eff} = \left(1+\frac{r}{m}\right)^m - 1$$