

Instructions: No books or notes may be used on this exam. Calculators are allowed, but not if they are on a cell phone or other communication device. You will have 2 hours to answer all of the following questions. Please write legibly and keep your paper as organized as possible. If you need more space on a question, then use the back of the page to continue your work and clearly indicate that you used the back on the front of the page. **If you do not indicate that you used the back, then your work on the back will not be graded.**

Show all your work! Answers without work or explanation may not receive full credit. Please use complete sentences where appropriate to explain your responses. You may leave answers with terms like

$$\binom{5}{2}, {}_5C_2, 17!, \frac{23}{41}, 15^4$$

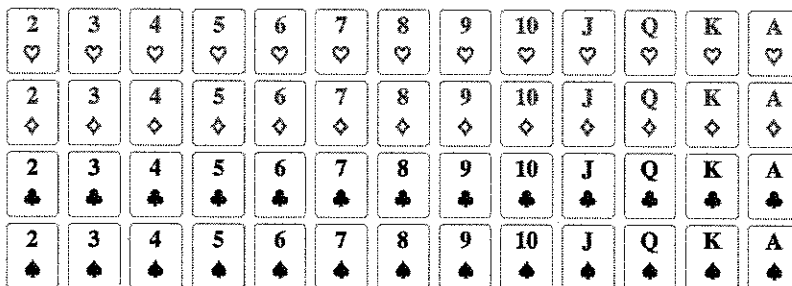
and simply providing a numerical answer from your calculator will not be sufficient for most problems. If your answer comes from your calculator, then I want to know what you entered into your calculator.

Good luck!

Name: Solutions A

Section: _____

Problem	Score	Possible
1		15
2		15
3		10
4		15
5		20
6		15
7		15
Total		100(+5)



(1) An experiment consists of rolling a 10-sided die.

(a) Write a set, S , that describes the sample space of this experiment.

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

(b) Are the events "an even number is rolled" and "a number less than 6 is rolled" mutually exclusive. Explain.

No. Rolling a 2 satisfies both and therefore both events could happen at the same time.

(c) Your friend tells you that the probability of an event, E , occurring in this experiment is 2. Is this possible? Explain.

No. The probability of an event occurring is always a number between 0 and 1.

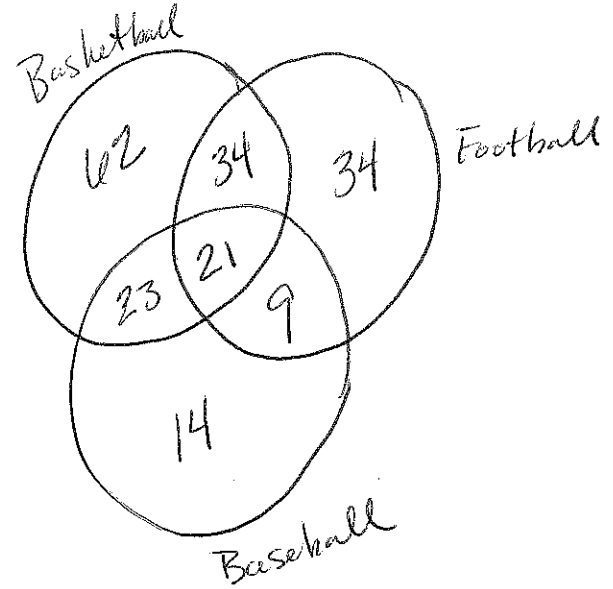


(2) A recent survey of 300 college students asked about the sports they watch on television. The results of the survey are as follows:

- 140 students watch basketball
- ✓98 students watch football
- ✓67 students watch baseball
- ✓55 students watch basketball and football
- ✓44 students watch basketball and baseball
- ✓30 students watch football and baseball
- ✓21 students watch all three

(a) How many students watch only baseball?

14



(b) How many students watch both basketball and football, but not baseball?

34

(c) How many students watch none of the three sports?

$$300 - (62 + 34 + 34 + 23 + 21 + 9 + 14) = 103$$

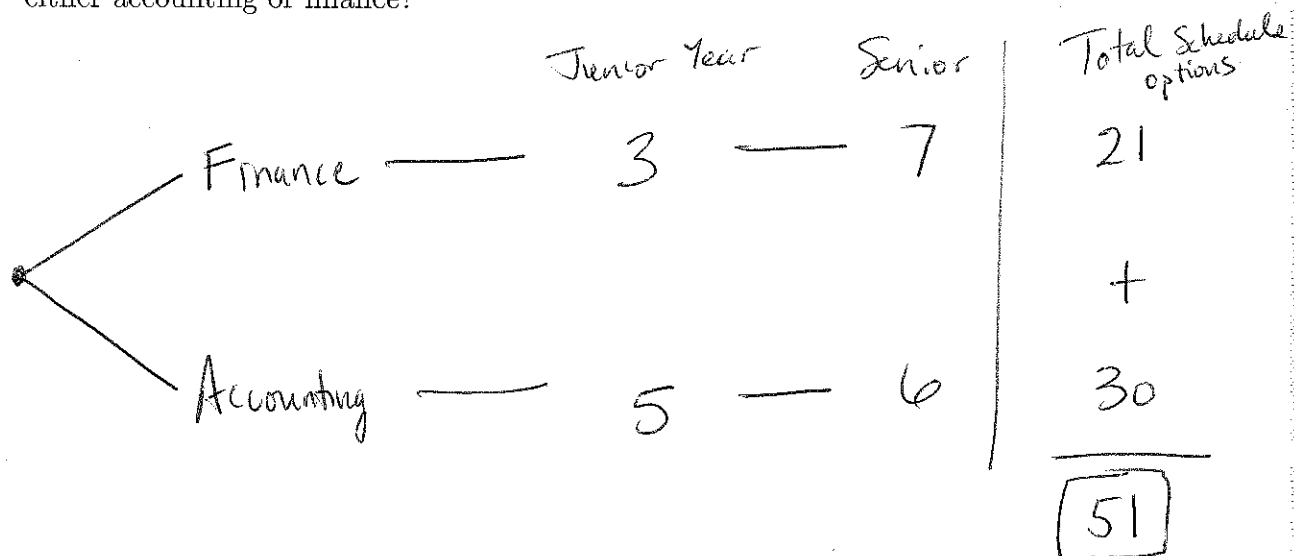


(3) Answer the following questions:

- (a) A senior is about to graduate and needs a few elective classes in her last semester. She can choose from 2 economics classes at 8am, 4 finance classes at 10am, 2 math classes at 12pm, and 3 miscellaneous other courses offered at 12pm. How many different ways can this student fill out these three time slots with the above classes?

$$\begin{array}{ccc} \text{8am classes} & \text{10am classes} & \text{12pm classes} \\ 2 & \cdot & 4 & \cdot & (2 + 3) \\ & & & & \\ & & & & = 2 \cdot 4 \cdot 5 = \boxed{40} \end{array}$$

- (b) Another student, a sophomore, is thinking ahead to plan for his last two years in college. He is considering majoring in accounting or finance. If he majors in accounting, then he has 5 different schedule options for his junior year each of which could be followed by 6 different schedule options for his senior year. If he majors in finance, then he has 3 different schedule options for his junior year each of which could be followed by 7 different schedule options for his senior year. How many total options does this student have to finish a degree in either accounting or finance?



- (4) Two cards are drawn (at the same time) from a standard deck of 52 cards.
 (a) What is the probability that at least one of the cards is red?

$$P(\text{at least one red}) = 1 - P(\text{no reds})$$

$$= 1 - \frac{\binom{26}{2}}{\binom{52}{2}}$$

- (b) What is the probability that one of the cards is a spade and one of the cards is a club?

$$\frac{\binom{13}{1} \binom{13}{1}}{\binom{52}{2}}$$

- (c) What is the probability that at least one card is a 9 or a heart?

← 36 such cards

$$1 - P(\text{no 9 and no heart})$$

$$1 - \frac{\binom{36}{2}}{\binom{52}{2}}$$



- (5) The graduating class of a high school has 150 members. Two different committees need to be formed.
- (a) How many ways could a prom committee of 10 students be chosen from the 150 member class?

$$\binom{150}{10}$$

- (b) How many ways could three members of the class be chosen as a class president, vice president, and secretary?

$$150 P_3 = 150 \cdot 149 \cdot 148$$

- (6) A group of 7 friends have tickets to a UK basketball game. Included in the 7 friends are two married couples and three single people (two male, one female).
- (a) How many ways can the friends sit in a row at the game if both married couples are to be seated together?

Consider each married couple as one unit.

ways to order the singles/couples \leftarrow $5! \cdot 2 \cdot 2$ \rightarrow # ways to swap other couple

\rightarrow # ways to swap one couple

- (b) How many ways can the friends sit in a row at the game if the men are to be seated together and the women are to be seated together?

$$4! \cdot 3! \cdot 2$$

Men Women

\rightarrow could switch whether men or women go first



(7) An exam consists of 10 true/false questions and 10 multiple choice questions with 3 answer choices for each multiple choice question. All questions are worth 5 points for a total of 100 points possible on the exam. A student randomly guesses on all questions.

(a) What is the probability that a student gets a 0 on the exam?

$$\frac{1^{10} \cdot 2^{10}}{2^{10} \cdot 3^{10}} = \boxed{\frac{1}{3^{10}}}$$

(b) What is the probability that a student gets 3 true/false questions and 7 multiple choice questions correct.

$$\frac{\binom{10}{3} 1^3 \cdot 1^7 \cdot \binom{10}{7} 1^7 \cdot 2^3}{2^{10} \cdot 3^{10}}$$

(c) What is the probability that the student gets a 95 or higher on the exam?

$P(100\%)$

$P(95\% \text{ missing one T/F})$

$P(95\% \text{ missing one MC})$

$$\frac{1^{10} \cdot 1^{10}}{2^{10} \cdot 3^{10}} + \frac{\binom{10}{1} 1^9 \cdot 1^1 \cdot 1^{10}}{2^{10} \cdot 3^{10}} + \frac{1^{10} \cdot 1^9 \cdot 2 \cdot \binom{10}{1}}{2^{10} \cdot 3^{10}}$$



(8) A bag contains 10 orange marbles, 11 black marbles, and 9 yellow marbles. You begin choosing marbles one at a time without replacement.

(a) What is the probability that you choose a black marble on the third pull given that you do not choose a black marble on either of the first two pulls?

$$P(\text{3rd Black} \mid \text{No black on 1st \& 2nd}) = \boxed{\frac{11}{28}}$$

(b) What is the probability that you choose a black marble on the third pull given that you choose a black marble on either the first or second pull, but not both?

$$P(\text{3rd Black} \mid \text{1 black on 1st or 2nd}) = \boxed{\frac{10}{28}}$$

(c) What is the probability that you choose a black marble on the third pull?

$$P(\text{3rd Black}) = P(\text{3rd Black} \mid \text{No black 1st \& 2nd}) P(\text{No black 1st \& 2nd}) \\ + P(\text{3rd Black} \mid \text{one black 1st or 2nd}) P(\text{1 black 1st or 2nd}) \\ + P(\text{3rd Black} \mid \text{two black 1st \& 2nd}) P(\text{two black 1st \& 2nd})$$

$$= \boxed{\left(\frac{11}{28}\right)\left(\frac{19}{30}\right)\left(\frac{18}{29}\right) + \left(\frac{10}{28}\right)\left(\frac{11}{30}\right)\left(\frac{19}{29}\right) \cdot 2 + \left(\frac{9}{28}\right)\left(\frac{11}{30}\right)\left(\frac{10}{29}\right)}$$



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Show all your work! Answers without work or explanation may not receive full credit. Please use complete sentences where appropriate to explain your responses. You may leave answers with terms like

$$\binom{5}{2}, {}_5C_2, 17!, \frac{23}{41}, 15^4$$

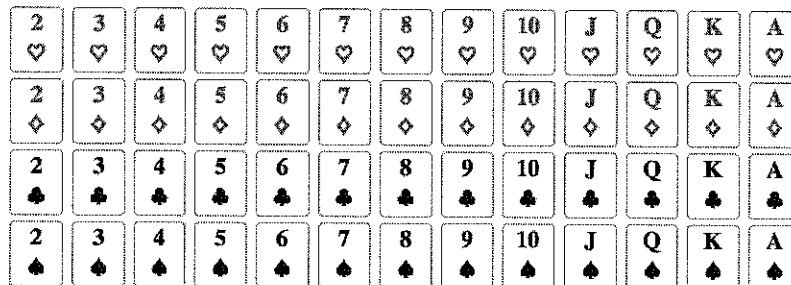
and simply providing a numerical answer from your calculator will not be sufficient for most problems. If your answer comes from your calculator, then I want to know what you entered into your calculator.

Good luck!

Name: Solutions B

Section: _____

Problem	Score	Possible
1		15
2		15
3		10
4		15
5		20
6		15
7		15
Total		100(+5)



(1) An experiment consists of rolling a 12-sided die.

(a) Write a set, S , that describes the sample space of this experiment.

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

(b) Are the events "an even number is rolled" and "a number less than 6 is rolled" mutually exclusive. Explain.

No. If a 2 gets rolled, then both events ~~are~~ occur so they can happen at the same time.

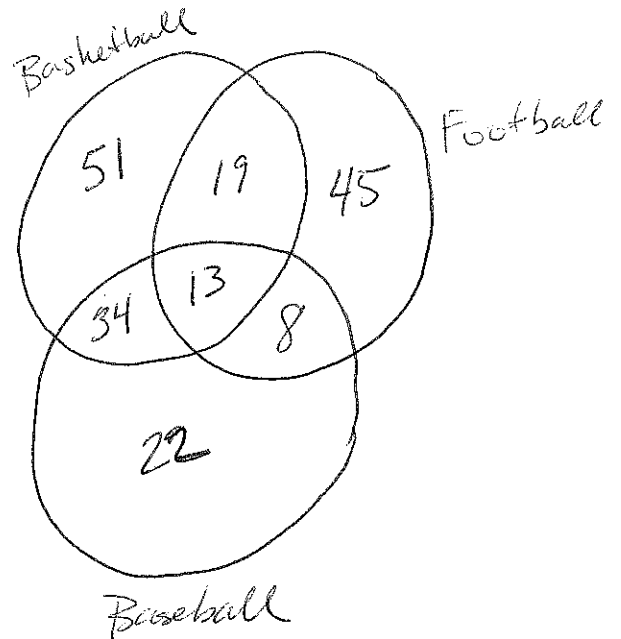
(c) Your friend tells you that the probability of an event, E , occurring in this experiment is 2. Is this possible? Explain.

No. Probabilities for events must be between 0 and 1.



(2) A recent survey of 300 college students asked about the sports they watch on television. The results of the survey are as follows:

- ✓ 117 students watch basketball
- ✓ 85 students watch football
- ✓ 77 students watch baseball
- ✓ 32 students watch basketball and football
- ✓ 47 students watch basketball and baseball
- ✓ 21 students watch football and baseball
- ✓ 13 students watch all three



(a) How many students watch only basketball?

51

(b) How many students watch both baseball and football, but not basketball?

8

(c) How many students watch none of the three sports?

$$300 - (51 + 19 + 45 + 34 + 13 + 8 + 22) = 108$$



(3) Answer the following questions:

- (a) A senior is about to graduate and needs a few elective classes in her last semester. She can choose from 3 economics classes at 8am, 3 finance classes at 10am, 2 math classes at 12pm, and 4 miscellaneous other courses offered at 12pm. How many different ways can this student fill out these three time slots with the above classes?

$$\begin{array}{ccc} 8\text{am} & 10\text{am} & 12\text{pm} \\ 3 & \cdot & 3 \cdot (2 + 4) = \boxed{54} \end{array}$$

- (b) Another student, a sophomore, is thinking ahead to plan for his last two years in college. He is considering majoring in accounting or finance. If he majors in accounting, then he has 4 different schedule options for his junior year each of which could be followed by 8 different schedule options for his senior year. If he majors in finance, then he has 5 different schedule options for his junior year each of which could be followed by 7 different schedule options for his senior year. How many total options does this student have to finish a degree in either accounting or finance?

	Junior	Senior	<u>Combined</u>
Accounting	4	8	32
			+
Finance	5	7	35
			<hr/>
			$\boxed{67}$



(4) Two cards are drawn (at the same time) from a standard deck of 52 cards.

(a) What is the probability that at least one of the cards is a club?

$$P(\text{at least one club}) = 1 - P(\text{no clubs})$$

$$= 1 - \frac{\binom{39}{2}}{\binom{52}{2}}$$

(b) What is the probability that one of the cards is red and one of the cards is black?

$$\frac{\binom{26}{1} \binom{26}{1}}{\binom{52}{2}}$$

(c) What is the probability that at least one card is a *J* or a diamond?

$$1 - P(\text{no } J \text{ and no diamond})$$

$$= 1 - \frac{\binom{36}{2}}{\binom{52}{2}}$$



(5) The graduating class of a high school has 225 members. Two different committees need to be formed.

(a) How many ways could a prom committee of 15 students be chosen from the 225 member class?

$$\binom{225}{15}$$

(b) How many ways could three members of the class be chosen as a class president, vice president, and secretary?

$$225 P_3 = 225 \cdot 224 \cdot 223$$

(6) A group of 7 friends have tickets to a UK basketball game. Included in the 7 friends are two married couples and three single people (all female).

(a) How many ways can the friends sit in a row at the game if both married couples are to be seated together?

Consider each married couple as one unit.

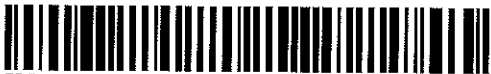
$$5! \cdot 2 \cdot 2$$

(b) How many ways can the friends sit in a row at the game if the men are to be seated together and the women are to be seated together?

$$2! \cdot 5! \cdot 2$$

men women

↑ could switch whether a man or woman goes first.



(7) An exam consists of 12 true/false questions and 8 multiple choice questions with 3 answer choices for each multiple choice question. All questions are worth 5 points for a total of 100 points possible on the exam. A student randomly guesses on all questions.

(a) What is the probability that a student gets a 0 on the exam?

$$\frac{1^{12} \cdot 2^8}{2^{12} \cdot 3^8} = \frac{1}{2^4 \cdot 3^8}$$

(b) What is the probability that a student gets 3 true/false questions and 5 multiple choice questions correct.

$$\frac{\binom{12}{3} 1^3 \cdot 1^9 \cdot \binom{8}{5} \cdot 1^5 \cdot 2^3}{2^{12} \cdot 3^8}$$

(c) What is the probability that the student gets a 95 or higher on the exam?

$P(100\%)$

$P(95\% \text{ missing one T/F})$

$P(95\% \text{ missing one MC})$

$$\frac{1}{2^{12} \cdot 3^8} + \frac{\binom{12}{11}}{2^{12} \cdot 3^8} + \frac{\binom{8}{7} \cdot 2}{2^{12} \cdot 3^8}$$



(8) A bag contains 8 orange marbles, 15 black marbles, and 7 yellow marbles. You begin choosing marbles one at a time without replacement.

- (a) What is the probability that you choose a black marble on the third pull given that you do not choose a black marble on either of the first two pulls?

$$P(\text{3rd Black} \mid \text{No black 1st or 2}) = \boxed{\frac{15}{28}}$$

- (b) What is the probability that you choose a black marble on the third pull given that you choose a black marble on either the first or second pull, but not both?

$$P(\text{3rd Black} \mid \text{1 Black on 1st + 2nd}) = \boxed{\frac{14}{28}}$$

- (c) What is the probability that you choose a black marble on the third pull?

$$\begin{aligned} P(\text{3rd Black}) &= P(\text{3rd Black} \mid \text{No black 1st or 2nd}) P(\text{No black 1st or 2nd}) \\ &\quad + P(\text{3rd Black} \mid \text{1 Black 1st or 2nd}) P(\text{1 Black 1st or 2nd}) \\ &\quad + P(\text{3rd Black} \mid \text{2 Black on 1st + 2nd}) P(\text{2 Black on 1st + 2nd}) \\ &= \boxed{\left(\frac{15}{28}\right)\left(\frac{15}{30}\right)\left(\frac{14}{29}\right) + \left(\frac{14}{28}\right)\left(\frac{15}{30}\right)\left(\frac{15}{29}\right) \cdot 2 + \left(\frac{13}{28}\right)\left(\frac{15}{30}\right)\left(\frac{14}{29}\right)} \end{aligned}$$

