MA 162 - Finite Mathematics

Exam #2

Fall 2014

October 20th, 2014

Instructions: No books or notes may be used on this exam. Calculators are allowed, but not if they are on a cell phone or other communication device. You will have 2 hours to answer all of the following questions. Please write legibly and keep your paper as organized as possible. If you need more space on a question, then use the back of the page to continue your work and clearly indicate that you used the back on the front of the page. **If you do not indicate that you used the back, then your work on the back will not be graded.**

Show all your work! Answers without work or explanation may not receive full credit. Please use complete sentences where appropriate to explain your responses. On multiple choice and true/false questions be sure to clearly indicate your answer choice. If there is any question about which answer you were trying to select then the question will be graded as incorrect.

Good luck!

Name: Solutions A

Section: All

Problem	Score	Possible
1		16
2		14
3		20
4		10
5		8
6		20
7		12
Total		100

1721

10

(1) (16 points)

(a) (8 points) Find the inverse (if it exists) of the matrix

$$\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$$

If the matrix does not have an inverse then explain why not.

Answer $\boxed{\begin{bmatrix} 2/7 & -\frac{1}{14} \\ 1/7 & 3/14 \end{bmatrix}}$

either ① $\left[\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ -2 & 4 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \to R_1 \cdot \frac{1}{3}} \left[\begin{array}{cc|cc} 1 & 1/3 & 1/3 & 0 \\ -2 & 4 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_2 \to R_2 + 2R_1} \left[\begin{array}{cc|cc} 1 & 1/3 & 1/3 & 0 \\ 0 & 14/3 & 2/3 & 1 \end{array}\right] \xrightarrow{R_2 \to \frac{3}{14} R_2} \left[\begin{array}{cc|cc} 1 & 1/3 & 1/3 & 0 \\ 0 & 1 & \frac{1}{7} & 3/14 \end{array}\right]$

$\xrightarrow{R_1 \to R_1 - \frac{1}{3} R_2} \left[\begin{array}{cc|cc} 1 & 0 & 2/7 & -1/14 \\ 0 & 1 & 1/7 & 3/14 \end{array}\right]$ inverse

OR ② use the formula

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(b) (8 points) The matrices

$$A^{-1} = \begin{bmatrix} \frac{-1}{5} & 0 & \frac{-3}{5} \\ \frac{3}{10} & 0 & \frac{2}{5} \\ \frac{-1}{15} & \frac{1}{3} & \frac{7}{15} \end{bmatrix} \text{ and } A = \begin{bmatrix} 4 & 6 & 0 \\ 5 & 4 & 3 \\ -3 & -2 & 0 \end{bmatrix}$$

are inverses. Use this information to solve the system of equations

$$\begin{array}{rcrcrcl} 4x & + & 6y & & & = & 50 \\ 5x & + & 4y & + & 3z & = & 18 \\ -3x & - & 2y & & & = & 45 \end{array}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 50 \\ 18 \\ 45 \end{bmatrix} = \begin{bmatrix} -1/5 & 0 & -3/5 \\ 3/10 & 0 & 2/5 \\ -1/15 & 1/3 & 7/15 \end{bmatrix}\begin{bmatrix} 50 \\ 18 \\ 45 \end{bmatrix}$$

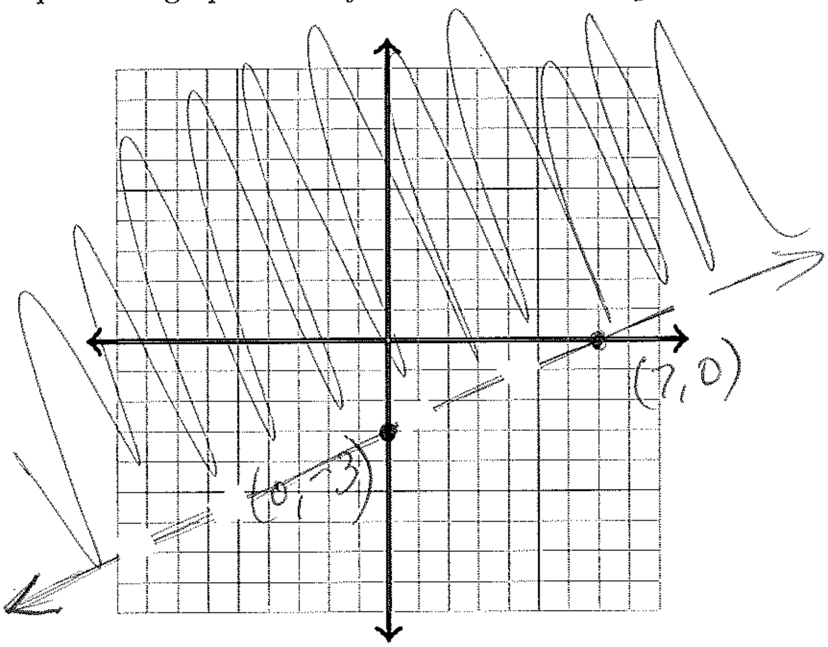
$$= \begin{bmatrix} -10 - 45 \\ 15 + 18 \\ -\frac{10}{3} + 6 + 21 \end{bmatrix} = \boxed{\begin{bmatrix} -55 \\ 33 \\ 71/3 \end{bmatrix}}$$

(2) (14 points)

(a) (8 points) Graph the solution set for the inequality

$$3x - 7y < 21$$

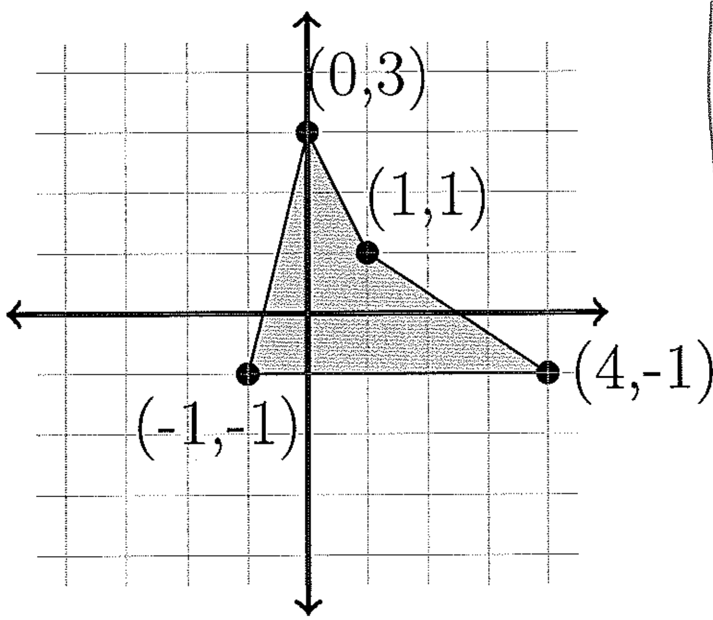
on the provided graph. Clearly indicate the intercepts.



line must be dashed

(b) (6 points) Below is the graph of the feasible region for a linear programming problem. The four labeled dots are the corner points. What is the minimum of the function $P = 5x - y$ on this feasible region?

x	y	P
0	3	-3
1	1	4
4	-1	21
-1	-1	-4



The minimum of P is -4 occurring at (-1,-1)

(3) (20 points) *Note: A complete answer to this problem must include clearly defined variables, objective function, and constraints; a clearly drawn and labeled feasible region on the provided axes; and the final answer in a complete sentence using the context of the problem.*

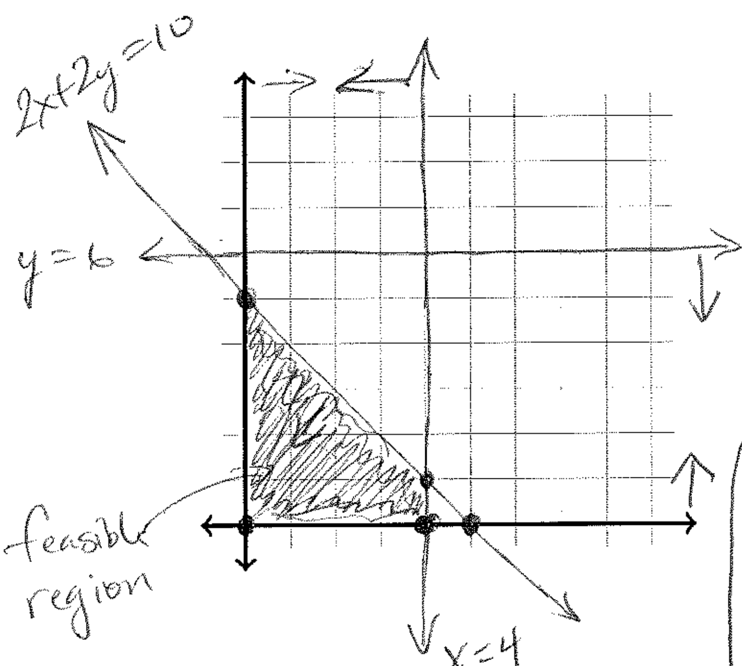
Steve has some leftover material in his woodworking shop that he has decided to use to make doghouses and benches to sell at the local craft fair. Steve wants to make at most 4 doghouses and at most 6 benches. He earns a profit of \$27 per doghouse and \$20 per bench. Making a doghouse takes Steve 2 hours and making a bench takes Steve 2 hours as well. Steve has 10 hours to build these items. Use the **method of corners** to find how many doghouses and how many benches Steve should make to maximize his profit.

Let x = # doghouses made
y = # benches made

Objective: maximize $P = 27x + 20y$

Constraints: $0 \le x \le 4$; $0 \le y \le 6$

$2x + 2y \le 10$



Corner pts	P
(0,5)	100
(0,0)	0
(4,0)	108
(4,1)	128

The maximum profit Steve can make is \$128 dollars when he makes 4 doghouses and 1 bench.

(4) (10 points) SETUP BUT DO NOT SOLVE the following linear programming problem. A complete answer to this problem must include clearly defined variables, objective function, and constraints. Do not setup a simplex table.

Farmer Frank has decided to plant corn, rice, and wheat in a 42-acre field he owns. Farmer Frank must use all 42 acres of the field. His farmhands are allowed to work at most a total of 8000 hours during the year. Each acre of corn requires 125 hours of work per year, each acre of rice requires 270 hours of work per year and each acre of wheat requires 65 hours of work per year. It costs him \$250 to plant one acre of corn, \$135 to plant one acre of rice, and \$275 to plant one acre of wheat. Farmer Frank also likes wheat more that corn and rice so he requires that the number of acres of wheat planted must be at least as much as the number of acres of corn and rice combined. How many acres of corn, rice, and wheat should Farmer Frank plant to minimize his cost?

Let x = # acres corn planted
y = # acres rice planted
z = # acres wheat planted

Objective: Minimize $C = 250x + 135y + 275z$

Constraints: $x + y + z = 42$

$125x + 270y + 65z \leq 8000$

$z \geq x + y$

$x \geq 0$

$y \geq 0$

$z \geq 0$

1721
14

(5) (8 points) A toy company wants to sell teddy bears and stuffed lions. They will use the Simplex Algorithm to determine how many of each type of stuffed animal they will make. They decide that x represents the number of teddy bears to make, y represents the number of stuffed lions to make, u represents the amount of stuffing leftover (in ft^3), v represents the number of unused plastic eyeballs, w represents the amount of thread leftover (in yards), and P represents the profit. The result of the Simplex Algorithm is the following matrix:

$$\left[\begin{array}{cccccc|c} x & y & u & v & w & P & RHS \\ \hline 0 & 1 & 1 & 7 & 0 & 0 & 100 \\ 1 & 1 & 0 & 3 & 0 & 0 & 215 \\ 0 & 5 & 0 & -2 & 1 & 0 & 35 \\ \hline 0 & 3 & 0 & 5 & 0 & 1 & 4300 \end{array}\right]$$

(a) How many of each type of stuffed animal should the company make?

215 teddy bears
0 stuffed lions

(b) What is the profit that the company will make?

\$4300

(c) How much of each supply is leftover at the end?

1000 ft^3 stuffing
0 eyeballs
35 yds. thread.

1721
15

(6) (20 points) The following table shows up in the middle of the Simplex Algorithm:

$$\left[\begin{array}{c|c|cccc|c} x & y & u & v & w & P & RHS \\ \hline 0 & 2 & 1 & 0 & -1 & 0 & 10 \\ 0 & 3 & 0 & 1 & 0 & 0 & 12 \\ 1 & -1 & 0 & 0 & 4 & 0 & 3 \\ 0 & -3 & 0 & 0 & 2 & 1 & 26 \end{array}\right]$$

Ratio
$10/2 = 5$
$12/3 = 4$

Answer the following questions:

(a) What are the non-basic variables?

y and w

(b) What is the next pivot position? Explain how you get this.

Choose column y b/c it is the only one with a negative in the bottom row

Choose row 2 b/c it has the smallest ratio. (Ignore negatives and zeros in the pivot column)

So the $\boxed{(2,2)\text{-entry}}$ is the next pivot.

(c) Carry out the pivot operation. Use the back of this page if you need extra space.

$$\xrightarrow{R_2 \to \frac{1}{3}R_2} \left[\begin{array}{cccccc|c} 0 & 2 & 1 & 0 & -1 & 0 & 10 \\ 0 & 1 & 0 & 1/3 & 0 & 0 & 4 \\ 1 & -1 & 0 & 0 & 4 & 0 & 3 \\ 0 & -3 & 0 & 0 & 2 & 1 & 26 \end{array}\right] \xrightarrow[\substack{R_3 \to R_3 + R_2 \\ R_4 \to R_4 + 3R_2}]{R_1 \to R_1 - 2R_2}$$

$$\begin{array}{c} \\ \left[\begin{array}{cccccc|c} 0 & 0 & 1 & -2/3 & -1 & 0 & 2 \\ 0 & 1 & 0 & 1/3 & 0 & 0 & 4 \\ 1 & 0 & 0 & 1/3 & 4 & 0 & 7 \\ 0 & 0 & 0 & 1 & 2 & 1 & 38 \end{array}\right] \end{array}$$

RHS

1721

16

(7) (12 points) Consider the following linear programming problem.

Minimize $C = 2x - y$ subject to

$$\begin{array}{rcrcr} x & + & y & \leq & 15 \\ x & - & 3y & \leq & 1 \\ 2x & + & y & \leq & 6 \end{array}$$

and $x \geq 0$, $y \geq 0$.

(a) How many slack variables will be used in the Simplex method?

3 (one for each inequality)

(b) Setup the initial table for the Simplex Algorithm.

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 15 \\ 1 & -3 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 1 & 0 & 6 \\ \hline 2 & -1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

(c) Explain in a sentence or two how you got the last row of your Simplex table.

First we must change from minimizing C to maximizing $P = -C = -2x + y$. Then subtract everything to the left side So the last row corresponds to

$2x - y + P = 0$.