MA 162: Finite Mathematics - Section 7.4

Fall 2014

Ray Kremer

University of Kentucky

With Correction

November 10, 2014

Announcements:

- Homework 7.3 due next Tuesday at 6pm.
- Homework 7.4 due Friday at 6pm.

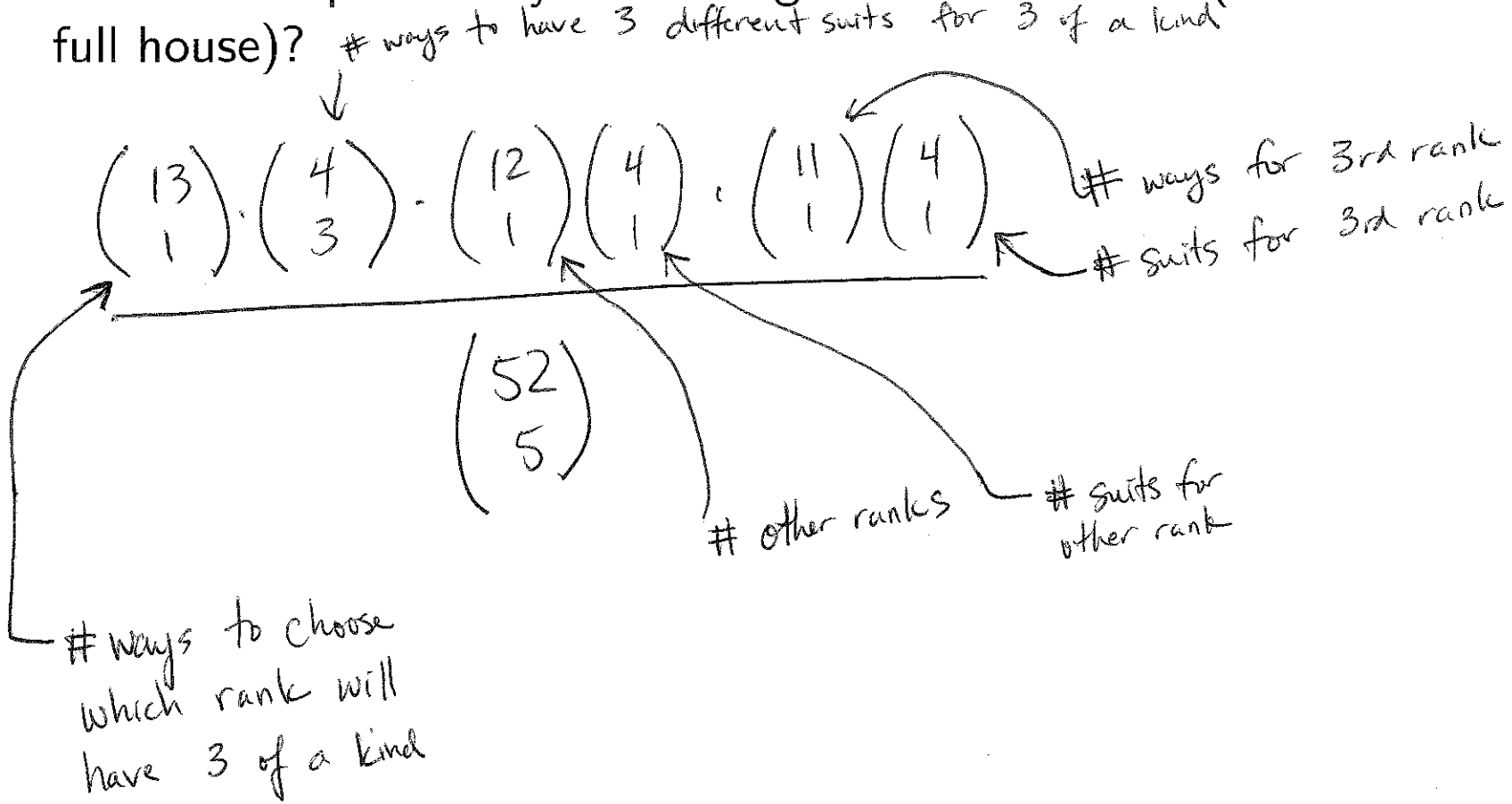
7.4 - Counting Techniques in Probability

- Let S be a uniform sample space (all outcomes equally likely).
- Let E be any event in S.
- To find the probability of E, find:
  - the number of outcomes in E
  - the number of possible outcomes in S
- Then divide to get

$$P(E) = \frac{n(E)}{n(S)}$$

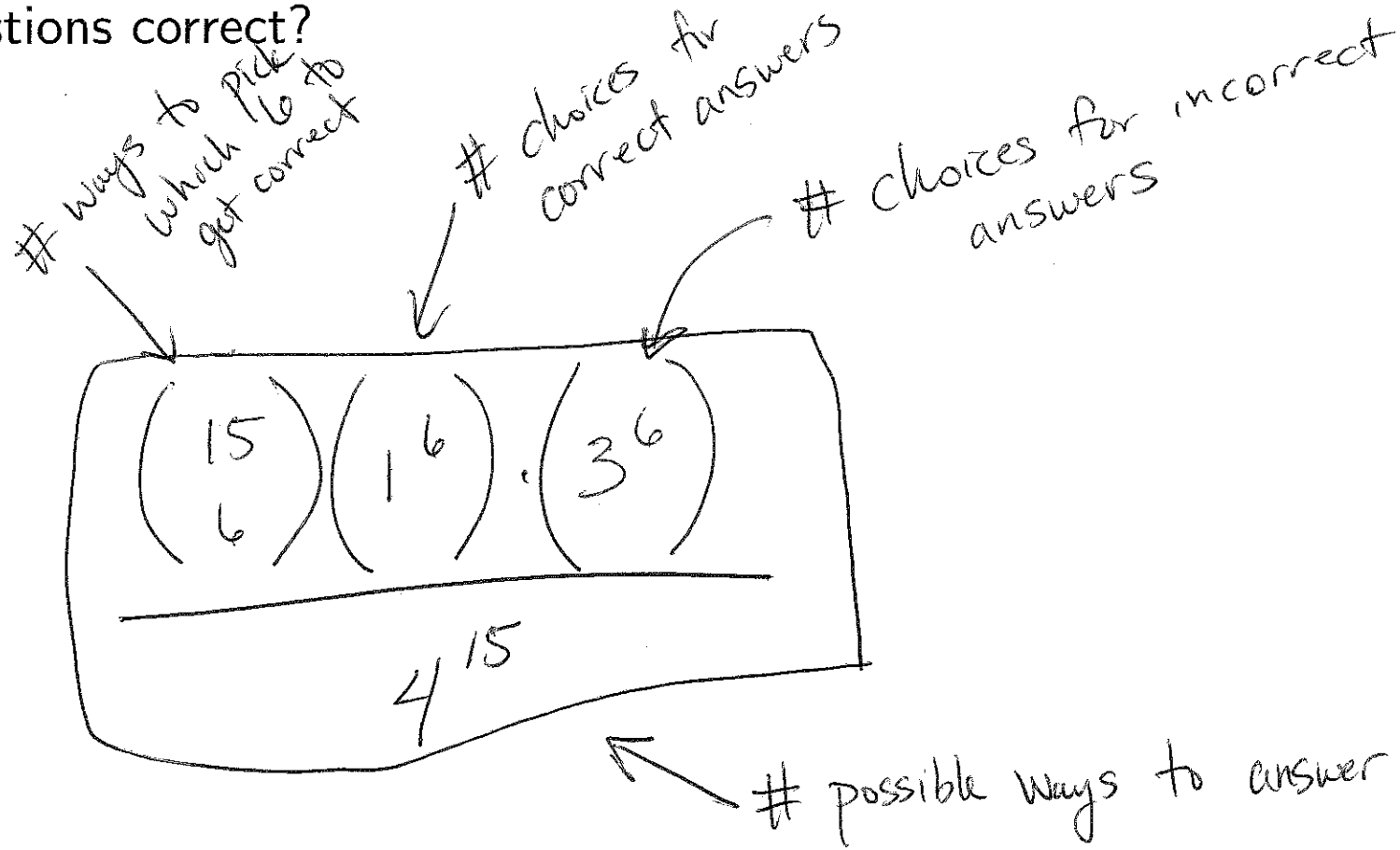
7.4 - Playing Cards

- A 5 card hand is drawn from a standard deck of 52 cards.
- What is the probability of drawing "three of a kind" (not a full house)?



7.4 - Multiple Choice Exam

- A math test consists of 15 multiple choice questions.
- Each question has 4 choices.
- One student forgot to study and decides to randomly guess on every question.
- What is the probability that the student gets exactly 6 questions correct?



7.4 - Multiple Choice Exam

- A math test consists of 15 multiple choice questions.
- Each question has 4 choices.
- One student forgot to study and decides to randomly guess on every question.
- What is the probability that the student gets an B (80% or higher) on the exam?

Answers 12 or more questions correct.

ways to get 12 correct | # ways to get 13 correct | # ways to get 14 correct | # ways to get 15 correct

$$\frac{\binom{15}{12}1^{12}\cdot 3^{3} + \binom{15}{13}1^{13}\cdot 3^{2} + \binom{15}{14}1^{14}\cdot 3^{1} + \binom{15}{15}1^{15}\cdot 3^{0}}{4^{15}}$$

7.4 - Defective Phones

A tech company manufactures cell phones and distributes them through the mail. The most recent batch of 1000 phones made has 20 defective phones in it.

- A large financial corporation orders 200 phones for its employees from this batch of phones. What is the probability that exactly 3 of the ordered phones are defective?

$$\frac{\binom{20}{3}\binom{980}{197}}{\binom{1000}{200}}$$

ways to choose 3 of 20 defective

ways to choose 197 of 980 non-defective

ways to choose 200 of 1000 total phones

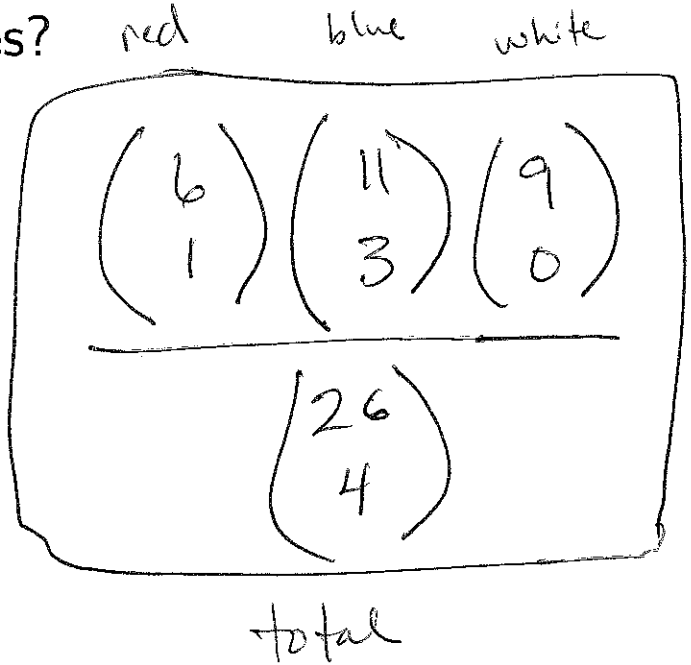
- What is the probability that at least one of the phones is defective?

$$1 - P(\text{no defective})$$

$$1 - \frac{\binom{980}{200}}{\binom{1000}{200}}$$

7.4 - Drawing Marbles

- A box contains 6 red marbles, 9 white marbles, and 11 blue marbles.
- You reach in and draw 4 marbles at the same time.
- What is the probability that you draw 1 red marble and 3 blue marbles?



7.4 - Drawing Marbles

- A box contains 6 red marbles, 9 white marbles, and 11 blue marbles.
- You reach in and draw marbles one at a time, without replacement.
  ↳ order matters
- What is the probability that you draw your first white marble on the 3rd pull?

P(drawing white on 3rd pull)

$$\frac{17}{26} \cdot \frac{16}{25} \cdot \frac{9}{24}$$

P(not getting a white on 1st pull)

P(not getting a white on 2nd pull)

Exploration - Conditional Probability with Playing Cards

- Two cards are drawn in succession (without replacement) out of a standard deck of 52 cards.
- What is the probability that the second card is a diamond, given that the first card drawn is not a diamond?

$\frac{13}{51}$ $\left(\left(\frac{39}{52}\right)\cdot\left(\frac{13}{51}\right)\right.$ is the probability of both 1st card not a diamond and 2nd card diamond

- What is the probability that the second card is a diamond, given that the first card is a diamond?

$\frac{12}{51}$ $\left(\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\right.$ is the probability of both 1st and 2nd cards being diamonds.

- What is the probability that the second card is a diamond?

$$P(\text{2nd diamond}) = P(\text{2nd diamond} \mid \text{1st diamond}) \cdot P(\text{1st diamond}) + P(\text{2nd diamond} \mid \text{1st not diamond}) \cdot P(\text{1st not diamond})$$

$$= \left(\frac{12}{51}\right)\left(\frac{13}{52}\right) + \left(\frac{13}{51}\right)\left(\frac{39}{52}\right)$$