

MA 162: Finite Mathematics

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Announcements:

- The next Web Assign assignment will be due after Thanksgiving.
- Read sections 1-3 of the supplemental financial math notes.

Financial Mathematics

- Interest Earned vs. Interest Rate
- Compound Interest
- Effective Rate of Interest
- Continuous Compounding

Interest Earned vs. Interest Rate

- Interest earned is the difference between the accumulated amount (after some time) and the initial amount invested.
- Suppose you invest \$3000 in an account today. One year from now, the value of the account is \$3138.
- Interest Earned:
- Interest Rate:

Compound Interest

- We saw last time that an investment of $\$P$ for t years which earns interest once per year accumulates to

$$A = P(1 + r)^t$$

- Interest earned one year gains interest the next year.
- The opposite scenario (when interest does not earn interest on itself) is called *simple interest*.

Different Compounding Frequencies

- Every example we've done so far has only been compounded once per year, but in practice different institutions apply interest to accounts at different frequencies (every month, week, day, etc...).
- How does increasing compounding frequency affect the amount of interest earned?
- Compare the following two options:
 - Investing \$2000 for one year at 5% APR compounded annually.
 - Investing \$2000 for one year at 5% APR compounded semiannually.

Different Compounding Frequency

- Investing \$2000 for one year at 5% APR compounded annually.
- Investing \$2000 for one year at 5% APR compounded semiannually.

Compound Interest

- Suppose $\$P$ is invested for t years at an annual interest rate of r per year compounded m times per year.
- Let $i = r/m$. i is the interest rate per period
- Let $n = mt$. n is the number of periods
- The accumulated value, A , is given by

$$A = P(1 + i)^n$$

- Equivalently,

$$A = P \left(1 + \frac{r}{m} \right)^{mt}$$

Example

- \$2500 is invested for 6 years at 2.8% APR compounded weekly. Determine the accumulated value at the end of the 6 years.

Third Principle of Financial Mathematics

Assume the APR of an account earning compound interest is positive. Given a fixed amount of principal invested in the account, the future value of the investment increases if at least one of the following occurs (and nothing else changes):

- (a.) the time invested increases
- (b.) the compounding frequency increase
- (c.) the nominal interest rate increases

Third Principle of Financial Mathematics (cont.)

Given a fixed future value of an investment in the account, the present value of the investment decreases if at least one of the following occurs (and nothing else changes):

- (a.) the time invested increase
- (b.) the compounding frequency increases
- (c.) the nominal interest rate increases

Comparing Interest Rates

- Just like only being able to compare monetary values that occur at the same time, we can only directly compare interest rates that have the same compounding frequency.
- Which is better?
 - 6% APR compounded monthly
 - 6.1% APR compounded quarterly

Comparing Interest Rates

- The easiest way to compare interest rates is to see what happens to \$1 under each given conditions.
- If you prefer formulas, then you may memorize that the effective rate of change is given by

$$r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

Continuous Compounding

Consider investing \$1 at an annual interest rate of 100%. The following table has the future value of this investment after one year with the given compounding frequencies:

m	Future Value
1	\$2
12	\$2.61
365	\$2.71
365(24)(60)	\$2.72

If \$ P is invested for t years at a continuously compounded interest rate of r , then the accumulated value is

$$A = Pe^{rt}$$

Example

Which is better?

- 5% compounded continuously
- 5.3% compounded daily

Example

You invest \$1000 in an account that earns 2% interest compounded monthly. Two years later, you add \$500 to the account. Three years after that you decide to move all the money from this account into a different account that earns 5% compounded continuously. How much money do you have 7 years after the original investment?