MA 162: Finite Mathematics - Section 7.3

Fall 2014

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Announcements:

- Homework 7.1/7.2 due Friday at 6pm.
- Homework 7.3 due next Tuesday at 6pm.

7.3 - Rules of Probability

(1) $P(E) \geq 0$ for any event E.

(2) $P(S) = 1$. (S is the entire sample space)

(3) If E and F are mutually exclusive, then $E \cap F = \phi$

$$P(E \cup F) = P(E) + P(F)$$

7.3 - Rules of Probability

(4) If E and F are any two events of an experiment, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

(5) (Complements) If E is an event of an experiment and E^c denotes the complement of E, then

$$P(E^c) = 1 - P(E) \quad (*)$$

everything not in E

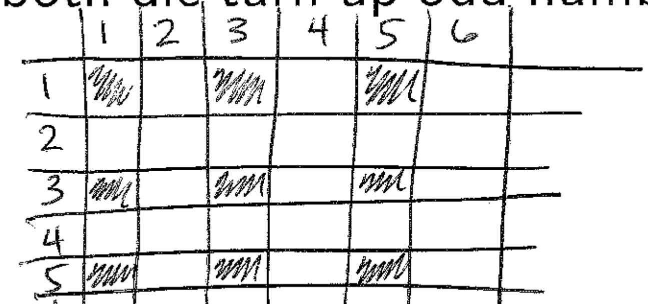
$$P(E^c) + P(E) = 1$$

Example - Rolling Dice

A pair of fair 6-sided dice are rolled.

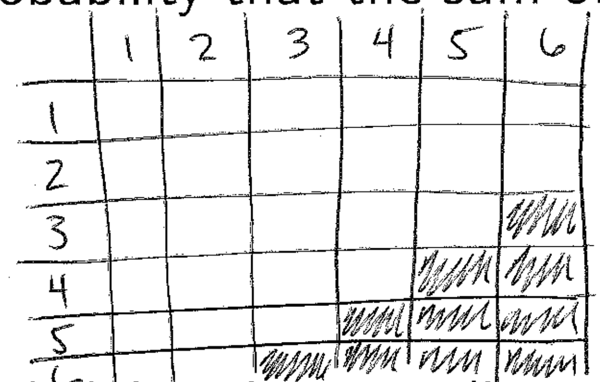
- Determine the probability that both die turn up odd numbers.

$$\frac{\text{\# ways to get both odd}}{\text{\# possible outcomes}}$$



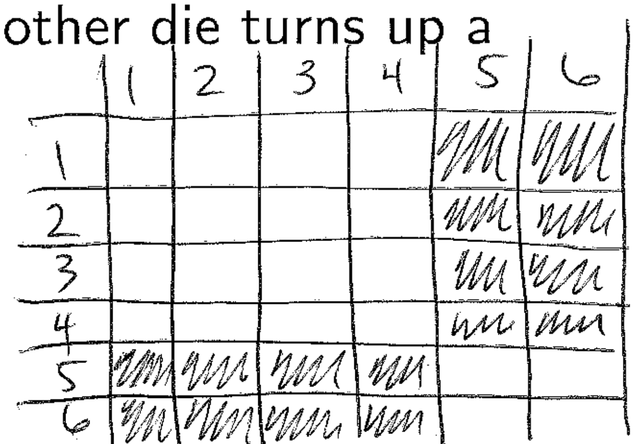
$$\frac{9}{36}$$

- Determine the probability that the sum of the values of the dice is at least 9.



$$\frac{10}{36}$$

- Determine the probability that one die turns up a number greater than or equal to 5 and the other die turns up a number less than or equal to 4.

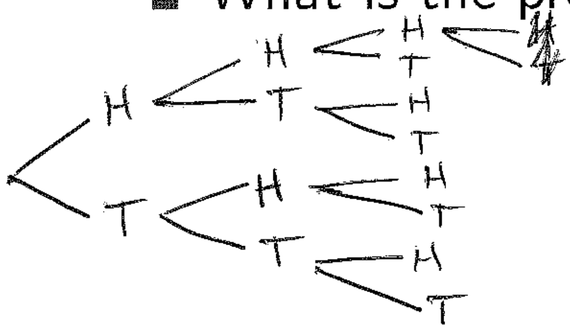


$$\frac{16}{36}$$

Example - Flipping Coins

A fair coin is flipped 4 times. 16 possible outcomes

- What is the probability of getting exactly 2 heads?



{HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

$\boxed{6/16}$

6 is really just $\binom{4}{2}$

- What is the probability of getting at least 2 heads?

$\boxed{\frac{11}{16}}$

Harder: P(exactly 2 H) + P(exactly 3 H) + P(exactly 4 H)

Complement → Easier: 1 − P(less than 2 H) = 1 − (P(0 H) + P(1 H))

$$1 - \left(\frac{1}{16} + \frac{4}{16}\right) = 1 - \frac{5}{16} = \frac{11}{16}$$

P(zero H) P(one H)

Example - Playing Cards

Two cards are drawn from a standard deck of 52 playing cards. (order does not matter)

- What is the probability that the two cards have the same suit?

$$\frac{\#\text{ ways to get 2 cards w/one suit}}{\#\text{ ways to choose 2 cards}} = \frac{\binom{4}{1}\cdot\binom{13}{2}}{\binom{52}{2}}$$

4 different ways to pick a suit

$\binom{13}{2}$ ways to pick two cards of the chosen suit

- What is the probability that at least one card has value from *J* up to *A*? Complement of choosing at least one card from J → A is choosing zero cards from J → A.

How many ways can you choose 2 cards w/ neither being b/w J + A?

$_{36}C_2$

Ans: $\boxed{1 - \frac{_{36}C_2}{_{52}C_2}}$

- What is the probability that the two card hand contains a Jack or a diamond?

$1 - P(\text{no Jack + no diamond})$

\# cards a Jack or diamond $= 4 + 13 - 1 = 16$

$$\boxed{1 - \left(\frac{_{36}C_2}{_{52}C_2}\right)}$$

Example - Playing Cards

Two cards are drawn from a standard deck of 52 playing cards.

- What is the probability that the two cards hand contains a Jack and a diamond?

$E =$ at least one card is a Jack

$F =$ at least one card is a diamond

Want $P(E \cap F)$; from last example $P(E \cup F) = 1 - \frac{{}_{36}C_2}{{}_{52}C_2}$

$$P(E \cap F) = P(E) + P(F) - P(E \cup F)$$

$$P(E) = 1 - P(\text{no Jack}) = 1 - \frac{{}_{48}C_2}{{}_{52}C_2}$$

$$P(F) = 1 - P(\text{no diamond}) = 1 - \frac{{}_{39}C_2}{{}_{52}C_2}$$

$$\boxed{P(\text{Jack} + \text{diamond}) = \left(1 - \frac{{}_{48}C_2}{{}_{52}C_2}\right) + \left(1 - \frac{{}_{39}C_2}{{}_{52}C_2}\right) - \left(1 - \frac{{}_{36}C_2}{{}_{52}C_2}\right)}$$

Example - Movie Survey

A group of 400 people aged 12 to 74 were surveyed about how often per month they go to a theater to see movies. The results are as follows:

Age ranges

\# movies per month seen

~~Age:~~	12 – 24	25 – 44	45 – 64	65 – 74	total
0	10	24	40	14	88
1 – 2	44	46	51	3	144
3 – 4	54	30	10	3	97
> 4	12	44	11	4	71
total	120	144	112	24	400

What is the probability that a random person from this survey...

- ...watches 0 movies per month? $88/400$
- ...watches at least 3 movies per month? $\frac{97+71}{400} = \frac{168}{400}$
- ...is less than 45 years old and watches between 1 and 4 movies per month? $\frac{44+46+54+30}{400} = \frac{174}{400}$

Example - Movie Survey

Age:	12 − 24	25 − 44	45 − 64	65 − 74	total
0	10	24	40	14	88
1 − 2	44	46	51	3	144
3 − 4	54	30	10	3	97
> 4	12	44	11	4	71
total	120	144	112	24	400

Conditional probability

- If you only consider those people in the 25 − 44 age range, what is the probability they watch at least 3 movies per month?

$$\frac{30+44}{144} = \frac{74}{144}$$

- If you only consider those people who watch at least 4 movies per month, what is the probability they are 25 − 64 years old?
(2nd & 3rd column)

$$\frac{44+11}{71} = \frac{55}{71}$$