MA 162: Finite Mathematics - Section 4.2

Fall 2014

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Announcements:

- Homework 4.1b due Tuesday at 6pm.
- Final Simplex Homework due ~~Friday~~ at 6pm.
  next Tuesday

The Simplex Algorithm

1. Setup the initial simplex tableau.

2. Determine whether the optimal solution has been reached by examining all entries in the last row to the left of the vertical line.

   - If all entries are non-negative, the optimal solution has been reached. Proceed to Step 4.
   - If there are one or more negative entries, the optimal solution has not been reached. Proceed to Step 3.

3. Perform the pivot operation. Locate the pivot element and convert that column to a unit column. Return to Step 2.

4. Determine the optimal solution(s). Non-basic variables get set to zero and the other variables can be read off the final table.

4.1 - Standard Maximization Problems

A **standard maximization problem** is one in which

- The objective function is to be maximized.
- All of the variables involved in the problem are non-negative.
- All other linear constraints may be written so that the expression involving the variables is less than or equal to a non-negative constant.

Tan, Section 4.1, #37

Justin has decided to invest up to \$60000 in medium-risk and high-risk stocks. The medium-risk stocks should make up at least 40% of the total investment, while the high-risk stocks should make up at least 20% of the total investment. He expects the medium-risk stocks will appreciate by 12% and the high-risk stocks will appreciate by 20% within a year. How much money should Justin invest in each type of stock to maximize the value of his investment?

Let x = amount invested in medium-risk stocks

y = " " " high-risk "

Objective: Maximize $P = .12x + .2y$

Subject to:

$$x + y \le 60000$$

$$x \ge .4(x+y) \longrightarrow 0 \ge -.6x + .4y$$

$$y \ge .2(x+y) \longrightarrow 0 \ge .2x - .8y$$

$$x \ge 0;\ y \ge 0$$

Initial Simplex Table

$$x + y + u = 60000$$
$$-.6x + .4y + v = 0$$
$$.2x - .8y + w = 0$$
$$-.12x - .2y + P = 0$$

$\left(\frac{\text{RHS}}{\text{column}}\right)$

$$\begin{array}{cccccc|c} x & y & u & v & w & P & \text{RHS} \\ \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 60000 \\ -.6 & .4 & 0 & 1 & 0 & 0 & 0 \\ .2 & -.8 & 0 & 0 & 1 & 0 & 0 \\ -.12 & -.2 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

Ratio
60000
0
—
—

4.2 - Minimization Problems with $\leq$ Constraints

A **minimization problem with $\leq$ constraints** is one in which

- The objective function, C, is to be minimized.
- All of the variables involved in the problem are non-negative.
- All other linear constraints may be written so that the expression involving the variables is less than or equal to a non-negative constant.

To minimize C, the objective function, we will maximize $-C$.

Tan, Section 4.2, #5

Minimize $C = 2x - 3y - 4z$ subject to

$$\begin{array}{rcrcrcr} -x & + & 2y & - & z & \leq & 8 \\ x & - & 2y & + & 2z & \leq & 10 \\ 2x & + & 4y & - & 3z & \leq & 12 \end{array}$$

and $x \geq 0$, $y \geq 0$, $z \geq 0$.

To minimize C is the same as maximizing $-C$.

Change the objective to maximizing $P = -C = -2x + 3y + 4z$

Add slacks: $-x + 2y - z + u = 8$

$$\begin{aligned} x - 2y + 2z + v &= 10 \\ 2x + 4y - 3z + w &= 12 \\ 2x - 3y - 4z + P &= 0 \end{aligned}$$

$$
\begin{array}{cccccccc}
x & y & z & u & v & w & p & \text{RHS} \\
\end{array}
$$

$$
\left[\begin{array}{ccccccc|c}
-1 & 2 & -1 & 1 & 0 & 0 & 0 & 8 \\
1 & -2 & 2 & 0 & 1 & 0 & 0 & 10 \\
2 & 4 & -3 & 0 & 0 & 1 & 0 & 12 \\
2 & -3 & -4 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\xrightarrow{R_2 \to \frac{1}{2}R_2}
$$

$$
\left[\begin{array}{ccccccc|c}
-1 & 2 & -1 & 1 & 0 & 0 & 0 & 8 \\
1/2 & -1 & 1 & 0 & 1/2 & 0 & 0 & 5 \\
2 & 4 & -3 & 0 & 0 & 1 & 0 & 12 \\
2 & -3 & -4 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\xrightarrow{\substack{R_1 \to R_1 + R_2 \\ R_3 \to R_3 + 3R_2 \\ R_4 \to R_4 + 4R_2}}
$$

$$
\left[\begin{array}{ccccccc|c}
-1/2 & 1 & 0 & 1 & 1/2 & 0 & 0 & 13 \\
1/2 & -1 & 1 & 0 & 1/2 & 0 & 0 & 5 \\
7/2 & 1 & 0 & 0 & 3/2 & 1 & 0 & 27 \\
4 & -7 & 0 & 0 & 2 & 0 & 1 & 20
\end{array}\right]
\quad
\begin{array}{c}
\text{Ratio} \\
13/1 = 13 \\
\text{———} \\
27/1 = 27 \\
\text{———}
\end{array}
$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 + 7R_1$

$$\longrightarrow \begin{array}{ccccccc|c} x & y & z & u & v & w & P & \text{RHS} \\ -\frac{1}{2} & 1 & 0 & 1 & \frac{1}{2} & 0 & 0 & 13 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 18 \\ 4 & 0 & 0 & -1 & 1 & 1 & 0 & 14 \\ \frac{1}{2} & 0 & 0 & 7 & \frac{11}{2} & 0 & 1 & 111 \end{array}$$

non-basic

$x = 0$, $u = 0$, $v = 0$

$y = 13 \quad z = 18 \quad w = 14 \quad P = 111$

Remember $P = -C$ so the minimum for C

is $\underline{\underline{-111}}$ which occurs at $(0, 13, 18)$.

Another Minimization Problem

Minimize $C = 2x + 3y + z$ subject to

$$\begin{array}{rcrcrcl} x & + & 3y & - & z & \leq & 18 \\ x & - & 2y & + & z & \leq & 11 \\ 4x & + & 2y & - & 3z & \leq & 25 \end{array}$$

and $x \geq 0$, $y \geq 0$, $z \geq 0$.

Change Objective to Maximize $P = -2x - 3y - z$.

Rewrite constraints:

$$\begin{aligned} x + 3y - z + u &= 18 \\ x - 2y + z + v &= 11 \\ 4x + 2y - 3z + w &= 25 \end{aligned}$$

Rewrite Objective $2x + 3y + z + P = 0$

$$
\begin{array}{ccccccc|c}
x & y & z & u & v & w & P & RHS \\
1 & 3 & -1 & 1 & 0 & 0 & 0 & 18 \\
1 & -2 & 1 & 0 & 1 & 0 & 0 & 11 \\
4 & 2 & -3 & 0 & 0 & 1 & 0 & 25 \\
2 & 3 & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

Non-basic ; $x=0$, $y=0$, $z=0$