MA 162: Finite Mathematics - Section 3.3

Fall 2014

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Announcements:

- Homework 3.1/3.2 due Friday at 6pm.
- Homework 3.3 due Tuesday at 6pm.

Linear Programming Problems Review

Remember that a linear programming problem consists of:

- An objective function (what are we trying to maximize/minimize?)
- Constraints (linear equalities or inequalities)

Today we will discuss how to find the optimal solution given the set of constraints in a graphical manner. Next week we will discuss how to do this entirely algebraically.

Terminology

- The set of all solutions to the system of constraints is called the **feasible set** or **feasible region**.
- Any point in the feasible region is called a **feasible solution**.
- Any point outside the feasible region is **not feasible** or **infeasible**.
- A point (if one exists) which optimizes (either maximizes or minimizes depending on the goal of the problem) the objective function is called an **optimal solution**.
- An optimal point must be feasible, but every feasible point is not necessarily optimal.

Theorem #1 - Solutions of Linear Programming Problems

If a linear programming problem has a solution, then it must occur at a vertex, or corner point, of the feasible set associated with the problem.

Furthermore, if the objective function is optimized at two adjacent vertices of the feasible region, then it is optimized at every point on the line segment joining these vertices, in which case there are infinitely many solutions to the problem.

Theorem #2 - Existence of a Solution

Suppose we are given a linear programming problem with a feasible set S and an objective function $P = ax + by$.

- If S is bounded, then P has both a maximum and a minimum value on S. A feasible set is bounded if you can enclose the entire set in a circle.
- If S is unbounded and both a and b are non-negative, then P has a minimum value on S provided that the constraints defining S include the inequalities $x \geq 0$ and $y \geq 0$.
- If S is the empty set, then the linear programming problem has no solution; that is, P has neither a maximum nor a minimum value.

The Method the Corners

1. Graph the feasible set.
2. Find the coordinates of all corner points (vertices) of the feasible set. Find the intersection of lines (possibly multiple times)
3. Evaluate the objective function at each corner point.
4. The optimal solution point is the point which produces the largest (or smallest) value found in the above step.

Solving LP Problems (Tan, Section 3.3, #42)

A farmer uses two types of fertilizers. A 50-lb bag of Fertilizer A contains 8 lb of nitrogen, 2 lb of phosphorus, and 4 lb of potassium. A 50-lb bag of Fertilizer B contains 5 lbs each of nitrogen, phosphorus, and potassium. The minimum requirements for a field are 440 lb of nitrogen, 260 lb of phosphorus, and 360 lb of potassium. If a 50-lb bag of Fertilizer A costs \$30 and a 50-lb bag of Fertilizer B costs \$20, find the amount of each type of fertilizer the farmer should use to minimize his cost while still meeting the minimum requirements.

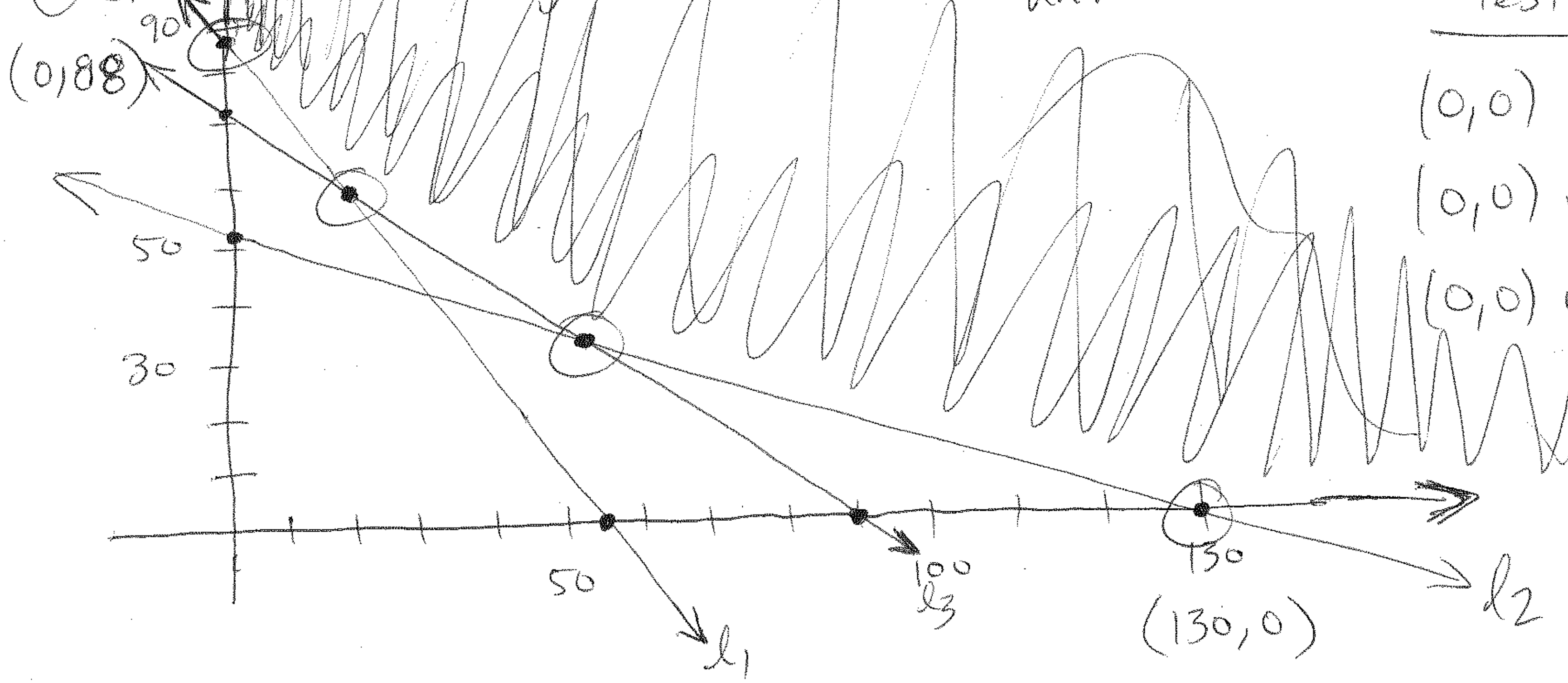
x = # of 50-lb bags of Fertilizer A
y = # of 50-lb bags of Fertilizer B

Obj: Minimize $C = 30x + 20y$

Constraints:			x-int	y-int
	$8x + 5y \geq 440$	l_1	$(55, 0)$	$(0, 88)$
$x \geq 0$	$2x + 5y \geq 260$	l_2	$(130, 0)$	$(0, 52)$
$y \geq 0$	$4x + 5y \geq 360$	l_3	$(90, 0)$	$(0, 72)$

① Graph Feasible Region

Unbounded



Test Pts

(0,0) in l_1 False

(0,0) in l_2 False

(0,0) in l_3 False

l_1 meets l_3:

$$\begin{aligned} 8x+5y &= 440 \\ 4x+5y &= 360 \\ \hline 4x &= 80 \\ x &= 20 \end{aligned}$$

$y = 56$

(20, 56)

l_2 meets l_3:

$$\begin{aligned} 4x+5y &= 360 \\ 2x+5y &= 260 \\ \hline 2x &= 100 \\ x &= 50 \end{aligned}$$

~~y = 36~~

$y = 32$

(50, 32)

x	y	C
0	88	1760
130	0	3900
20	56	1720
50	32	2140

The minimum cost meeting all the requirements is $1720 which occurs when the farmer buys 20 bugs of Fertilizer A and 56 bags of Fertilizer B.

Solving LP Problems (Tan, Section 3.3, #36)

A financier plans to invest up to $500,000 in two projects. Project A yields a return of 10% on the investment, whereas Project B yields a return of 15% on the investment. Because the investment in Project B is riskier than the investment in Project A, the financier has decided that the investment in Project B should not exceed 40% of the total investment. How much should she invest in each project to maximize the return on her investment? What is the maximum return?

We will begin with this example on Monday 10/6.