MA 162: Finite Mathematics - Section 6.1/6.2

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Announcements:

- Homework 6.1/6.2 due Tuesday at 6pm.

6.1 - Sets

- A **set** is a well-defined collection of objects. Well-defined just means that if you are given an object, you can tell if that belongs in the set or not.

- How do we write sets?

6.1 - Sets and Elements

- The things in a set are called **elements** (or *members*).
- Notation:
  - If x is an element of A, then we write $x \in A$.
  - If x is not an element of A, then we write $x \notin A$.

6.1 - Sets - Some More Definitions

- Two sets are **equal** if they have the same elements.
    - $\{1,2,3\}=\{3,2,1\}$ - The order of elements does not matter.

- If every element of a set A is also in the set B, then A is a **subset** of B.
    - $\{1,2\} \subset \{1,2,3\}$.

- The set that contains no elements is called the **empty set** and is denoted by $\emptyset$.

- Often there will be a large set which contains everything that might be of interest in a problem. This is called a **universal set**, usually denoted U.

6.1 - Set Operations

Let A and B be sets.

- Union - The **union** of A and B, written $A \cup B$, is the set of all elements that are in A or in B.
- Intersection - The **intersection** of A and B, written $A \cap B$, the set of all elements that are in A and in B.
    - The sets A and B are called **disjoint** if $A \cap B = \emptyset$.
- Complement - If U is a universal set and A is a subset of U, then the set of all elements in U that are not in A is called the **complement** of A, denoted A^c.

6.1 - Example

Suppose we roll a 6-sided die. The set of possible outcomes is $U = \{1, 2, 3, 4, 5, 6\}$.

- $A = \{2, 4, 6\}$=the even outcomes
- $B = \{3, 6\}$=multiples of 3
- $A \cup B =$
- $A \cap B =$
- $A^c =$
- $B^c =$

6.1 - De Morgan's Laws

- Let A and B be sets. Then
  - $(A \cup B)^c = A^c \cap B^c$
  - $(A \cap B)^c = A^c \cup B^c$
- Still using $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$, and $B = \{3, 6\}$ find the following:
  - $(A \cup B)^c =$
  - $A^c \cap B^c =$

6.1 - Why do we care?

- We use sets to solve some problems known as **counting problems**.

  Let's do some counting:

- How many distinct letters are in the word "balloon"? List these letters as the elements of a set A.

- How many distinct letters are in the word "dragon"? List these letters as the elements of a set B.

- How many distinct letters are there if we can take from either "balloon" or "dragon"?

6.2 - Inclusion-Exclusion

- We denote the number of elements in a set A by $n(A)$.
- In the last example we saw that $n(A \cup B) \neq n(A) + n(B)$.
- The only time $n(A \cup B) = n(A) + n(B)$ is if A and B are ____________________.
- In general,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

6.2 - Two Set Inclusion-Exclusion Example

On a certain day, the Wilton County Jail held 190 prisoners accused of a crime (felony and/or misdemeanor). Of these, 130 were accused of felonies and 121 were accused of misdemeanors. How many prisoners were accused of both a felony and a misdemeanor?

6.2 - Three Set Inclusion-Exclusion

- There is an equation similar to the equation $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for any number of finite sets.

- For three sets this is:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) \\ - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

6.2 - Three Set Inclusion-Exclusion Example

To help plan the number of meals (breakfast, lunch, and dinner) to be prepared in a college cafeteria, a survey was conducted and the following data were obtained:

- 130 students ate breakfast
- 180 students ate lunch
- 275 students ate dinner
- 68 students ate breakfast and lunch
- 112 students are breakfast and dinner
- 90 students ate lunch and dinner
- 58 students ate all three meals

How many students ate...

- ... at least one meal in the cafeteria?
- ... exactly one meal in the cafeteria?
- ... only dinner in the cafeteria?
- ... exactly two meals in the cafeteria?