# MA 162: Finite Mathematics - Section 6.3

Fall 2014

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Announcements:

- Homework 6.1/6.2 due Tuesday at 6pm.
- Homework 6.3 due Friday at 6pm.
- Exam Grades are posted on Web Assign. The actual exams will be returned later this week.

# 6.2 - Three Set Inclusion-Exclusion

- There is an equation similar to the equation $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ for any number of finite sets.

- For three sets this is:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) \\ - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

# 6.2 - Three Set Inclusion-Exclusion Example

To help plan the number of meals (breakfast, lunch, and dinner) to be prepared in a college cafeteria, a survey was conducted and the following data were obtained:

- 130 students ate breakfast
- 180 students ate lunch
- 275 students ate dinner
- 68 students ate breakfast and lunch
- 112 students are breakfast and dinner
- 90 students ate lunch and dinner
- 58 students ate all three meals

How many students ate...

- ... at least one meal in the cafeteria?
- ... exactly one meal in the cafeteria?
- ... only dinner in the cafeteria?
- ... exactly two meals in the cafeteria?

# 6.3 - The Multiplication Principle

- The next counting technique we will discuss is the multiplication principle.

- Anthony has 4 shirts and 2 ties currently in his closet. How many different shirt/tie combinations could Anthony choose from when he gets dressed the next time?

# 6.3 - The Multiplication Principle

- **The Multiplication Principle** - Suppose there are $m$ ways of performing a task $T_1$ and $n$ ways of performing a task $T_2$. Then there are $mn$ ways of performing the task $T_1$ followed by the task $T_2$.

- In the previous example, we saw that there were 8 possible shirt/tie combinations which is exactly $4 \cdot 2$.

# 6.3 - The Multiplication Principle

- But what if not every shirt goes with every tie?
- Anthony's roommate Billy has 4 shirts ($S_1$, $S_2$, $S_3$, $S_4$) and 3 ties ($T_1$, $T_2$, $T_3$) in his closet. However, $S_1$ only matches with $T_1$ and $S_2$ only matches with $T_2$ and $T_3$. How many possible shirt/tie combinations does Billy have?

# 6.3 - Generalized Multiplication Principle

- How many outcomes are possible if you flip a two-sided coin twice? Assume that flipping a heads first, then tails is different than tails first, then heads.

# 6.3 - Generalized Multiplication Principle

- How many outcomes are possible if you flip a two-sided coin three times? Assume the order matters again.

# 6.3 - Generalized Multiplication Principle

- **Generalized Multiplication Principle** - Suppose a task $T_1$ can be performed in $N_1$ ways, a task $T_2$ can be performed in $N_2$ ways, ..., and , finally, a task $T_m$ can be performed in $N_m$ ways. Then, the number of ways of performing the tasks $T_1$, $T_2$, ..., $T_m$ in succession is given by the product

$$N_1 N_2 \cdots N_m$$

- (Example, Tan, Section 6.3 #30) A rolling combination four-digit padlock is unlocked by moving each of four rollers so as to produce the correct sequence. Each roller has ten digits.

  (a) How many possible combinations are there?

# 6.3 - Generalized Multiplication Principle

- (Example, Tan, Section 6.3 #30) A rolling combination four-digit padlock is unlocked by moving each of four rollers so as to produce the correct sequence. Each roller has ten digits.
  (b) How many combinations are possible if no digit is repeated?

# 6.3 - Generalized Multiplication Principle

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  - Another way to ask this question:

# 6.3 - Generalized Multiplication Principle

- (Example, Tan, Section 6.3 #30 (modified)) A rolling combination four-digit padlock is unlocked by moving each of four rollers so as to produce the correct sequence. Each roller has four digits.
  - How many combinations are possible if no digit is repeated?

# Tan, Section 6.3, #17

- The Panini Cafe has a special lunch menu in which the diner chooses a soup or salad, a main course, and a dessert. There are three choices of soups, three choices of salad, five choices for the main course, and four choices for desserts. How many different three-course meals can diners order for lunch?