MA 162: Finite Mathematics - Section 4.1

Fall 2014

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Announcements:

- Homework 4.1a due Friday at 6pm.
- Homework 4.1b due Tuesday at 6pm.

The Simplex Algorithm

1. Setup the initial simplex tableau.
2. Determine whether the optimal solution has been reached by examining all entries in the last row to the left of the vertical line.
   - If all entries are non-negative, the optimal solution has been reached. Proceed to Step 4.
   - If there are one or more negative entries, the optimal solution has not been reached. Proceed to Step 3.
3. Perform the pivot operation. Locate the pivot element and convert that column to a unit column. Return to Step 2.
4. Determine the optimal solution(s). Non-basic variables get set to zero and the other variables can be read off the final table.

Tan, Section 4.1, #18

objective

Maximize $P = 5x + 3y$ subject to $\qquad -5x - 3y + P = 0$

Constraints
$$\begin{aligned} x + y &\leq 80 \\ 3x &\leq 90 \end{aligned} \qquad \begin{aligned} x + y + u &= 80 \\ 3x + v &= 90 \end{aligned}$$

and $x \geq 0$, $y \geq 0$.

$$\begin{array}{cccccc} x & y & u & v & P & \text{RHS} \end{array}$$
$$\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 80 \\ 3 & 0 & 0 & 1 & 0 & 90 \\ -5 & -3 & 0 & 0 & 1 & 0 \end{array}\right]$$

Ratio (RHS/Column Entry)

$80/1 = 80$

$90/3 = 30$ pivot row

pivot column

Now make the pivot column a unit column with a 1 in the pivot position.

$$\xrightarrow{R_2 \to \frac{1}{3}R_2} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 80 \\ 1 & 0 & 0 & 1/3 & 0 & 30 \\ -5 & -3 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_3 \to R_3 + 5R_2]{R_1 \to R_1 - R_2}$$

$$
\begin{array}{l|cccccc}
 & x & y & u & v & P & \text{RHS} \\
\hline
\text{pivot row} & 0 & 1 & 1 & -\tfrac{1}{3} & 0 & 50 \\
 & 1 & 0 & 0 & \tfrac{1}{3} & 0 & 30 \\
 & 0 & -3 & 0 & \tfrac{5}{3} & 1 & 150
\end{array}
$$

pivot column

Q: What corner point does this matrix correspond to?

A: $x = 30$
$y = 0$
$u = 50$
$v = 0$
$P = 150$

$\boxed{(30, 0)}$

$$
\xrightarrow{R_3 \to R_3 + 3R_1}
\begin{array}{cccccc}
x & y & u & v & P & \text{RHS}
\end{array}
$$

$$
\left[\begin{array}{ccccc|c}
0 & 1 & 1 & -\tfrac{1}{3} & 0 & 50 \\
1 & 0 & 0 & \tfrac{1}{3} & 0 & 30 \\
0 & 0 & 3 & \tfrac{2}{3} & 1 & 300
\end{array}\right]
$$

non-basic columns
($u = 0$ + $v = 0$)

$x = 30$
$y = 50$
$P = 300$

$\boxed{\text{P is maximized at } (30, 50) \text{ and has maximum value of } 300.}$

Tan, Section 4.1, #25

Maximize $P = 3x + 4y + 5z$ subject to $-3x - 4y - 5z + P = 0$

$$\begin{array}{rcrcrcl} x & + & y & + & z & \leq & 8 \\ 3x & + & 2y & + & 4z & \leq & 24 \end{array}$$

$x + y + z + u = 8$

$3x + 2y + 4z + v = 24$

and $x \geq 0$, $y \geq 0$, $z \geq 0$.

$$\begin{array}{ccccccc|c} x & y & z & u & v & P & \text{RHS} & \text{Ratio} \end{array}$$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 8 \\ 3 & 2 & 4 & 0 & 1 & 0 & 24 \\ -3 & -4 & -5 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} 8/1 = 8 \\ 24/4 = 6 \text{ pivot row} \\ \end{array}$$

pivot column

$\xrightarrow{R_2 \to \frac{1}{4}R_2}$

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 8 \\ 3/4 & 1/2 & 1 & 0 & 1/4 & 0 & 6 \\ -3 & -4 & -5 & 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_3 \to R_3 + 5R_2]{R_1 \to R_1 - R_2}$$

$$\begin{array}{c} \\ \\ \\ \end{array}
\begin{array}{ccccccc}
x & y & z & u & v & P & RHS \\
\end{array}$$

$$\left[\begin{array}{cccccc|c}
\frac{1}{4} & \frac{1}{2} & 0 & 1 & -\frac{1}{4} & 0 & 2 \\
\frac{3}{4} & \frac{1}{2} & 1 & 0 & \frac{1}{4} & 0 & 6 \\
\frac{3}{4} & -\frac{3}{2} & 0 & 0 & \frac{5}{4} & 1 & 30
\end{array}\right]
\quad
\begin{array}{c}
\text{Ratio} \\
2/(\tfrac{1}{2}) = 4 \\
6/(\tfrac{1}{2}) = 12 \\
\end{array}$$

pivot column

$$\xrightarrow{R_1 \to 2R_1}$$

$$\left[\begin{array}{cccccc|c}
\frac{1}{2} & 1 & 0 & 2 & -\frac{1}{2} & 0 & 4 \\
\frac{3}{4} & \frac{1}{2} & 1 & 0 & \frac{1}{4} & 0 & 6 \\
\frac{3}{4} & -\frac{3}{2} & 0 & 0 & \frac{5}{4} & 1 & 30
\end{array}\right]
\xrightarrow[R_3 \to R_3 + \frac{3}{2}R_1]{R_2 \to R_2 - \frac{1}{2}R_1}$$

$$\begin{array}{ccccccc}
x & y & z & u & v & P & RHS \\
\end{array}$$

$$\left[\begin{array}{cccccc|c}
\frac{1}{2} & 1 & 0 & 2 & -\frac{1}{2} & 0 & 4 \\
\frac{1}{2} & 0 & 1 & -1 & \frac{1}{2} & 0 & 4 \\
\frac{3}{2} & 0 & 0 & 3 & \frac{1}{2} & 1 & 36
\end{array}\right]$$

$y = 4$
$z = 4$
$P = 36$

Non-basic

$x = 0, \ u = 0, \ v = 0$

$\boxed{\text{P is maximized at } (0,4,4) \text{ and has a maximum value of } 36.}$

Tan, Section 4.1, #46

Boise Lumber manufactures prefabricated houses. They offer three models: standard, deluxe, and luxury. Each house is prefabricated and partially assembled in a factory. The final assembly is done on-site.
The dollar amount of building material required, the amount of labor required, the amount of on-site labor required, and the profit per unit are as follows:

	Standard	Deluxe	Luxury
Material	\$6000	\$8000	\$10000
Factory Labor	240	220	200
On-site Labor	180	210	300
Profit	\$3400	\$4000	\$5000

They have \$8200000 is budgeted for building materials, 218000 hours of factory labor, and 237000 hours of on-site labor.
How many houses of each type should they build in order to maximize their profit?

Let $x =$ # Standard houses made

$y =$ # deluxe houses made

$z =$ # luxury houses made

Maximize $P = 3400x + 4000y + 5000z$

Subject to: $6000x + 8000y + 10000z \le 8200000$ (slack u)

$240x + 220y + 200z \le 218000$ (slack v)

$180x + 210y + 300z \le 237000$ (slack w)

$x \ge 0, y \ge 0, z \ge 0$

<u>Intial Table:</u>

$$\begin{array}{cccccccc}
x & y & z & u & v & w & P & \text{RHS} \\
\end{array}$$

$$\left[\begin{array}{ccccccc|c}
6000 & 8000 & 10000 & 1 & 0 & 0 & 0 & 8200000 \\
240 & 220 & 200 & 0 & 1 & 0 & 0 & 218000 \\
180 & 210 & 300 & 0 & 0 & 1 & 0 & 237000 \\
-3400 & -4000 & -5000 & 0 & 0 & 0 & 1 & 0
\end{array}\right]$$