MA 162 Quiz 2 Thursday 11$^{\text{th}}$ September 2014

Problem 1

Determine whether each system of linear equations has a unique solution, infinitely many solutions, or no solution. Find all solutions whenever they exist.

a) $\begin{aligned} x - 3y &= -1 \\ 4x + 3y &= 11 \end{aligned}$ **b)** $\begin{aligned} 2x - 3y &= 6 \\ 6x - 9y &= 12 \end{aligned}$

a) Rewrite the two equations into the form y = mx + b

$$\begin{aligned} y &= (1/3)x + (1/3) \\ y &= -(4/3)x + (11/3). \end{aligned}$$

Substitute the first equation into the second and solve for x

$$\begin{aligned} (1/3)x + (1/3) &= -(4/3)x + (11/3) \\ x + 1 &= -4x + 11 \\ 5x + 1 &= 11 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

Substitute the value of x into one of the equations to obatin the corresponding y-coordinate.

$$y(2) = (2/3) + (1/3) = 3/3 = 1.$$

The system of equations have a unique solution at the point $\boxed{(2, 1)}$

b) Rewrite the two equations into the form y = mx + b

$$\begin{aligned} y &= (2/3)x - 2 \\ y &= (2/3)x - (4/3). \end{aligned}$$

Notice that both equations have the same slope, but different y-intercepts. This implies that the two lines are PARALLEL AND DISTINCT. Hence there are $\boxed{\text{NO SOLUTIONS}}$. Another way to obtain the answer is to substitute one equation into the other

$$\begin{aligned} (2/3)x - 2 &= (2/3)x - (4/3) \\ -2 &= (4/3). \end{aligned}$$

Which is clearly not true. The contradiction implies that there are NO SOLUTIONS

Problem 2

Determine whether the statement below is true or false. Explain you reasoning.
Suppose a system of equations is represented by two parallel lines. Then the system is *guaranteed* to have no solution
The assertion is false. If the two lines are PARALLEL AND COINCIDENT then there are infinitely many solutions.